International Journal of Applied Power Engineering (IJAPE)
Vol.7, No.2, August 2018, pp. 139~146
ISSN: 2252-8792 DOI: 10.11591/ijape.v7.i2.pp139-146

Analysis of Six-Phase Interior Permanent Magnet Synchronous Machines for Optimal Parameter Considerations

Tanmoy Dey[1], Amit Kumar Chowdhury[2], Sk Mehboob Alam[3], Surajit Mondal[4]
[1,2,3] Department of Electrical Engineering, Indian Institute of Engineering Science and Technology, Shibpur, India
[4] Department of Research and Development, University of Petroleum and Energy Studies, Dehradun, India

Article Info

Article history:

Received Apr 14, 2018
Revised Apr 25, 2018
Accepted May 12, 2018

Keyword:

IPMSM
Maximum torque per ampere control
Vehicles

ABSTRACT

Understanding the merits of six-phase interior permanent magnet synchronous machines (IP-MSMs) over their three-phase counterparts, this paper analyses the six-phase machine for optimal parameter and performance considerations. Initially, a mathematical model of the six-phase IPMSM is developed employing the dq-axis theory and performance predicted by the model is verified under identical operating conditions with those using a machine designed and tested through finite element analysis (FEA). The developed and verified machine model is then employed to exclusively derive the relation between various machine parameters in order to obtain optimum flux weakening region in the six-phase IPMSM. Thereafter, the equations derived on the basis of maximum torque per ampere (MTPA) control theory are analyzed to understand the effect of various parameters and variables in influencing the machine's performance in the 'constant torque' region and 'constant power' region, power output capability, a ratio of reluctance torque to magnet-assisted torque with changes in the stator current vector etc. This is the contribution of this paper.

Corresponding Author:

Surajit Mondal,
Department of Research and Development,
University of Petroleum and Energy Studies, Dehradun, India.
Email: surajitmondalee@gmail.com

1. INTRODUCTION

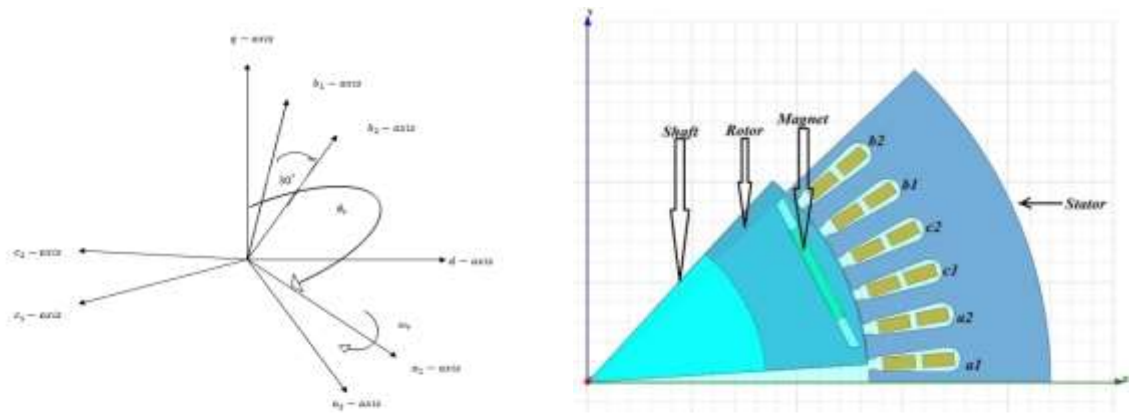
Over the last decade, research on six-phase machines have been widely performed and been seriously looked into as contenders for electric ship propulsion, locomotive traction, industrial high-power applications, electric and hybrid-electric vehicles and "more-electric" aircrafts. Fault tolerance feature, possibility of splitting the motor power across a higher number of phases and thus reducing the per-phase (per switch) converter rating, reduction of amplitude of torque pulsation, lesser harmonic content and more flexibility of torque/power control when compared to standard three-phase versions have been inherent merits of these machines [1-5]. Six phase conventional synchronous generators were modeled and developed for high power generation, and multi-phase induction and permanent magnet machines and drives were modeled, designed and tested for or industrial automation and electrified transportation applications as well [6]. Some of the paper also there for detecting defects by applying Hough transform and least squares on ceramic images obtained from non-destructive testing [7].

However, optimal parameter considerations in six-phase IPMSMs for optimal design and understanding its power capability have not been analyzed yet. This is where the authors derive their motivation from.

2. MATHEMATICAL MODEL OF THE SIX-PHASE IPMSM

Mathematical model of the six-phase IPMSM employing the dq axis theory has been developed in this section. The six-phase machine configuration taken into consideration employs two sets of balanced sinusoidal distributed three-phase windings, mutually displaced in space by 30 degrees electrical, and is shown in Figure 1. The derived model d-q axis model neglects the insignificant mutual leakage flux between the two stator winding sets, motoring mode of the machine is assumed positive in the analysis and the machine is analyzed in the rotor reference frame using Park's transformation as in [8].

The corresponding d- and q-axis flux linkage matrix can be obtained as in (1). The torque of the machine as in (2) can be derived using the flux linkage and voltage equations, where, sub-script 1 and 2 in various d- and q-axis quantities correspond to two three-phase sets a1b1c1 phases and a2b2c2 phases respectively. λ m is the peak value of the open-circuit permanent magnet flux linkage associated with one armature phase in the machine. The results obtained from both FEA and developed dq axis model were found to be in close agreement.



(a) (b)

Figure 1. Six-phase IPMSM stator phase configuration considered and cross-section of the six-phase IPMSM designed in this paper for FEA based validation (a) Six-phase IPMSM stator phase configuration (b) Cross-section of the six-phase IPMSM designed

To get the flux linkage and voltage equation, the phase inductances have to be transformed to d-q reference frame with a transformation matrix. Here we assumed that the same space and time difference between phases then the transformation matrix will look like following. [9]

$$M = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) & 0 & 0 & 0 \\ \sin\theta_r & \sin(\theta_r - 120^\circ) & \sin(\theta_r + 120^\circ) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta_r + 30^\circ) & \cos(\theta_r - 90^\circ) & \cos(\theta_r + 150^\circ) \\ 0 & 0 & 0 & \sin(\theta_r + 30^\circ) & \sin(\theta_r - 90^\circ) & \sin(\theta_r + 150^\circ) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

We know the procedure to transform the phase inductance to d-q inductances

$$\left[L_{q1d1\gamma1q2d2\gamma2}\right] = [M][L_{a1b1c1a2b2c2}][M^{-1}]$$

So the inductance matrix in dq frame will look like,

$$\begin{bmatrix} L_{q1} \\ L_{d1} \\ L_{\gamma 1} \\ L_{q2} \\ L_{d2} \\ L_{\gamma 2} \end{bmatrix} = \begin{bmatrix} L_q & 0 & 0 & L_{mq} & 0 & 0 \\ 0 & L_d & 0 & 0 & L_{md} & 0 \\ 0 & 0 & L_l & 0 & 0 & 0 \\ L_{mq} & 0 & 0 & L_q & 0 & 0 \\ 0 & L_{md} & 0 & 0 & L_d & 0 \\ 0 & 0 & 0 & 0 & 0 & L_l \end{bmatrix}$$

Where,

$$L_q = L_{ls} + \frac{3}{2}L_A - \frac{3}{2}L_B$$

$$L_d = L_{ls} + \frac{3}{2}L_A + \frac{3}{2}L_B$$

$$L_{mq} = \frac{3}{2}L_A - \frac{3}{2}L_B$$

$$L_{md} = \frac{3}{2}L_A + \frac{3}{2}L_B$$

Now the flux linkage matrix will form as below,

$$\begin{bmatrix} \lambda_{q1} \\ \lambda_{d1} \\ \lambda_{\gamma 1} \\ \lambda_{q2} \\ \lambda_{d2} \\ \lambda_{\gamma 2} \end{bmatrix} = \begin{bmatrix} L_q & 0 & 0 & L_{mq} & 0 & 0 \\ 0 & L_d & 0 & 0 & L_{md} & 0 \\ 0 & 0 & L_l & 0 & 0 & 0 \\ L_{mq} & 0 & 0 & L_q & 0 & 0 \\ 0 & L_{md} & 0 & 0 & L_d & 0 \\ 0 & 0 & 0 & 0 & 0 & L_l \end{bmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{\gamma 1} \\ i_{q2} \\ i_{d2} \\ i_{\gamma 2} \end{bmatrix} + \lambda_f \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Where, λ_f represents the permanent magnet flux linkage.

Finally the voltage equation will be,

$$v_{d1} = r_s i_{d1} + \frac{d\lambda_{d1}}{dt} - \omega\lambda_{q1} \tag{1}$$

$$v_{q1} = r_s i_{q1} + \frac{d\lambda_{q1}}{dt} + \omega\lambda_{d1} \tag{2}$$

$$v_{d2} = r_s i_{d2} + \frac{d\lambda_{d2}}{dt} - \omega\lambda_{q2} \tag{3}$$

$$v_{q2} = r_s i_{q2} + \frac{d\lambda_{q2}}{dt} + \omega\lambda_{d2} \tag{4}$$

Where,

$$\lambda_{d1} = L_d i_{d1} + L_{md} i_{d2} + \lambda_f \tag{5}$$

$$\lambda_{d2} = L_d i_{d2} + L_{md} i_{d1} + \lambda_f \tag{6}$$

$$\lambda_{q1} = L_q i_{q1} + L_{mq} i_{q2} \tag{7}$$

$$\lambda_{q2} = L_q i_{q2} + L_{mq} i_{q1} \tag{8}$$

To find out the Torque expression following steps need to be performed.
Expanding all four voltage equation and multiplied with corresponding current we can get the power expression,

$$v_{d1}i_{d1} = r_s{i_{d1}}^2 + L_d i_{d1}\frac{di_{d1}}{dt} + L_{md}i_{d1}\frac{di_{d2}}{dt} - \omega_r L_q i_{q1} i_{d1} - \omega_r L_{mq} i_{q2} i_{d1} \quad (9)$$

$$v_{q1}i_{q1} = r_s{i_{q1}}^2 + L_q i_{q1}\frac{di_{q1}}{dt} + L_{mq}i_{q1}\frac{di_{q2}}{dt} + \omega_r L_d i_{q1} i_{d1} + \omega_r L_{md} i_{q1} i_{d2} + \omega_r \lambda_f i_{q1} \quad (10)$$

$$v_{d2}i_{d2} = r_s{i_{d2}}^2 + L_d i_{d2}\frac{di_{d2}}{dt} + L_{md}i_{d2}\frac{di_{d1}}{dt} - \omega_r L_q i_{q2} i_{d2} - \omega_r L_{mq} i_{q1} i_{d2} \quad (11)$$

$$v_{q2}i_{q2} = r_s{i_{q2}}^2 + L_q i_{q2}\frac{di_{q2}}{dt} + L_{mq}i_{q2}\frac{di_{q1}}{dt} + \omega_r L_d i_{q2} i_{d2} + \omega_r L_{md} i_{d1} i_{q2} + \omega_r \lambda_f i_{q2} \quad (12)$$

Integrating (9), (10), (11) and (12) with respect to time and adding them gives the energy balance equation as:

$$\begin{aligned}&\int\left(v_{d1}i_{d1} + v_{q1}i_{q1} + v_{d2}i_{d2} + v_{q2}i_{q2}\right)dt = \int\left(r_s{i_{d1}}^2 + r_s{i_{q1}}^2 + r_s{i_{d2}}^2 + r_s{i_{q2}}^2\right)dt + \\ &\int\begin{pmatrix} L_d i_{d1} di_{d1} + L_{md} i_{d1} di_{d2} + L_q i_{q1} di_{q1} + L_{mq} i_{q1} di_{q2} + L_d i_{d2} di_{d2} \\ +L_{md} i_{d2} di_{d1} + L_q i_{q2} di_{q2} + L_{mq} i_{q2} di_{q1}\end{pmatrix} dt + \\ &\int \omega_r \begin{pmatrix} L_d i_{q1} i_{d1} + L_{md} i_{q1} i_{d2} + L_d i_{q2} i_{d2} + L_{md} i_{d1} i_{q2} - L_q i_{q1} i_{d1} \\ -L_{mq} i_{q2} i_{d1} - L_q i_{q2} i_{d2} - L_{mq} i_{q1} i_{d2} + \lambda_f i_{q2} + \lambda_f i_{q1}\end{pmatrix} dt \end{aligned} \quad (13)$$

The first term represents the Ohmic losses and the second term represents the stored field energy in the coils and the last term is responsible to produce useful electromagnetic torque.
So finally the electromagnetic torque expression is,

$$T_e = \frac{3}{2} \times \frac{polepairs}{\omega_r} \times \frac{dW_{em}}{dt}$$

Where, dW_{em} represents the change in mechanical energy, i.e. last part of expression (13),

$$T_e = \frac{3}{2} \times \frac{p}{2} \times \begin{bmatrix} \lambda_f\left(i_{q1} + i_{q2}\right) + \left(L_d - L_q\right)\left(i_{q1} i_{d1} + i_{q2} i_{d2}\right) \\ +\left(L_{md} - L_{mq}\right)\left(i_{q1} i_{d2} + i_{d1} i_{q2}\right)\end{bmatrix} \quad (14)$$

Where, p= no of pole-pairs.

From the previous voltage equations we get steady state form of voltage equations like:

$$V_{d1} = r_s I_{d1} - \omega L_q I_{q1} - \omega L_{mq} I_{q2} \quad (15)$$

$$V_{q1} = r_s I_{q1} + \omega L_d I_{d1} + \omega L_{md} I_{d2} + \omega \lambda_f \quad (16)$$

$$V_{d2} = r_s I_{d2} - \omega L_q I_{q2} - \omega L_{mq} I_{q1} \quad (17)$$

$$V_{q2} = r_s I_{q2} + \omega L_d I_{d2} + \omega L_{md} I_{d1} + \omega \lambda_f \quad (18)$$

2.1. Equivalent circuit

Equivalent circuit for six phase IPMSM (a) d-axis, (b) q-axis shown in Figure 2.

(a) (b)

Figure 2. Equivalent circuit for six phase IPMSM (a) d-axis, (b) q-axis

At steady state the voltage equations will be for the a1b1c1 winding set from the (15), (16), (17) and (18)

$$V_{d1} = r_s I_{d1} - \omega L_q I_{q1} - \omega L_{mq} I_{q2} = r_s I_{d1} - \omega \lambda_{d1} \tag{19}$$

$$V_{q1} = r_s I_{q1} + \omega L_d I_{d1} + \omega L_{md} I_{d2} + \omega \lambda_f = r_s I_{q1} + \omega \lambda_{q1} \tag{20}$$

The stator current vector and voltage vector may be defined as

$$\overrightarrow{I_a} = I_{d1} + jI_{q1} \tag{21}$$

$$\overrightarrow{V_a} = V_{d1} + jV_{q1} \tag{22}$$

From (15), (16), (17), (18), (21) and (22) we can write

$$\begin{aligned} \overrightarrow{V_a} &= r_s I_{d1} - \omega \lambda_{d1} + J\left(r_s I_{q1} + \omega \lambda_{q1}\right) \\ &= r_s\left(I_{d1} + JI_{q1}\right) + J\omega\left(\lambda_{q1} + J\lambda_{d1}\right) \\ &= r_s I_a + j\omega \lambda_1 \end{aligned} \tag{23}$$

Where we consider,

$$\overrightarrow{\lambda_1} = \lambda_{d1} + j\lambda_{q1} \tag{24}$$

And I_a and V_a are the magnitudes of the stator current and voltage respectively.

With the help of the above equations, the phasor diagram has been drawn in the figures 3 and 4. We consider that the stator voltage $\overrightarrow{V_a}$ and stator current $\overrightarrow{I_a}$ are leading the q axis at angles of γ and β respectively in fig: here the d-axis current component is demagnetized the permanent magnet field mmf, and also we draw a phasor where $\overrightarrow{V_a}$ leads the q-axis by γ and the current phasor $\overrightarrow{I_a}$ lags the q axis by β in fig: that's why in the second phasor d-axis current component magnetizing the permanent magnet field mmf.

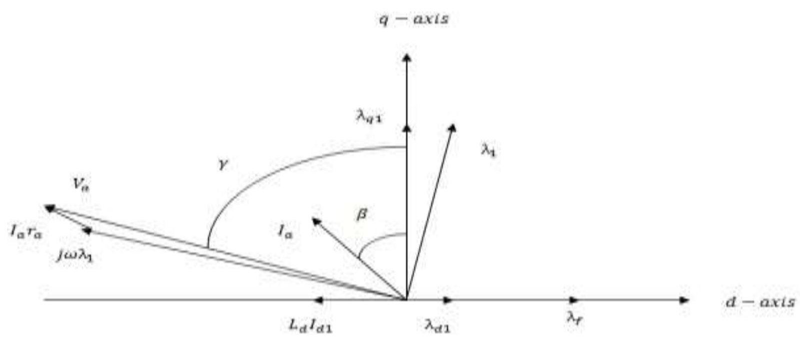


Figure 3. Steady-state phasor diagram for IPMSM when demagnetizing armature current in the d-axis

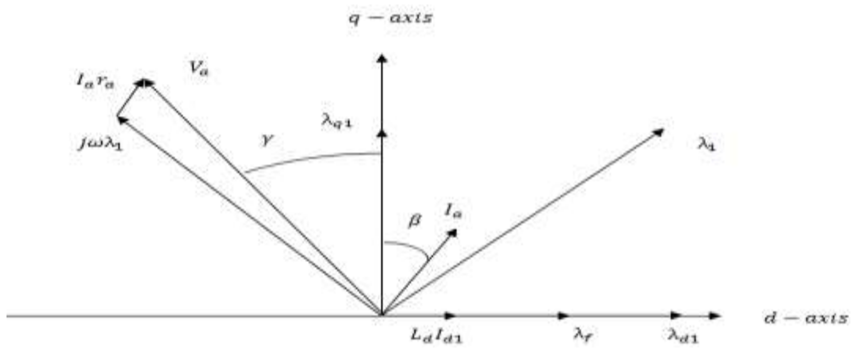


Figure 4. Steady-state phasor diagram for IPMSM when magnetizing armature current in the d-axis

3. DERIVATION OF THE MTPA BASED OPTIMUM POWER CAPABILITY CONDITION

The developed dq-axis model presented in the previous section will be employed here to exclusively derive the Maximum Torque per Ampere (MTPA) control theory which will establish a basis towards understanding the relationship between various machine parameters in order to obtain optimal torque control below rated (base) speed and flux weakening capability above base speed in 6-phase IPMSMs. The MTPA theory established here for a six-phase IPMSM is one of the contributions of this paper. As per the conventional dq-axis theory, a) the machine is assumed to have sinusoidal distributed MMF's both from the stator and the rotor side, b) core loss, mechanical loss, air-gap space harmonics and magnetic saturation are neglected. From the mathematical model presented in section II, d- and q- axis voltage equations of the six-phase machine can be written as in (3) and (4), where other symbols have their usual meanings.

$$v_d = r_s i_d + \frac{d\lambda_d}{dt} - \omega\lambda_q \tag{25}$$

$$v_q = r_s i_q + \frac{d\lambda_q}{dt} + \omega\lambda_d \tag{26}$$

Where,

$$\lambda_d = (L_d + L_{md})i_d + \lambda_m$$
$$\lambda_q = (L_q + L_{mq})i_q$$

It is assumed that three phase balanced "30^0 time-phase displaced" currents are fed through the two symmetrical three-phase sets such that $i_{d1} = i_{d2} = i_d$, $i_{q1} = i_{q2} = i_q$, $v_{d1} = v_{d2} = v_d$, and $v_{q1} = v_{q2} = v_q$.

$$T_e = \frac{6p}{4} \times \left[\lambda_m i_q + 2\left(L_d - L_q\right) i_d i_q\right] \tag{27}$$

The torque equation can be written as in (5). Where, p is the differential operator, ω is the electrical speed in rad/sec (other symbols having their usual meanings). Again, 1) considering steady state analysis of the machine; and 2) for simplicity of analysis, neglecting resistive voltage drops.

The d- and q-axis voltage equations of the six-phase machine can be written as in (28) and (29). The per phase peak voltage Va can be written as in (30). Substituting (28) and (29) in (30) we get (31).

$$V_q = \omega_r(\lambda_m + (2L_d - L_{ls})) \tag{28}$$

$$V_d = -\omega_r I_q\left(2L_q - L_{ls}\right) \tag{29}$$

$$V_a{}^2 = V_d{}^2 + V_q{}^2 \tag{30}$$

$$\left(I_q(2L_q - L_{ls})\right)^2 + \left((\lambda_m + (2L_d - L_{ls}))\right)^2 I_q{}^2 = \left(\frac{V_a}{\omega}\right)^2 \tag{31}$$

$$I_a{}^2 = I_d{}^2 + I_q{}^2 \tag{32}$$

If the d- and q-axes currents are considered as abscissa and ordinate variables respectively, equation (31) can be graphically represented through an ellipse whose center is not at the origin and has co-ordinates of ellipse center as$(((-\lambda_m/2L_d - L_{ls}\,, 0),0)$. Equation (32) presents a circle and the peak of current per phase (Ia) can be represented as its radius. The graphical representations of the voltage and current in the form of ellipse and circle, respectively, are in similar lines with the MTPA concept for three-phase IPMSMs [10].

4. ANALYSIS OF THE MTPA BASED EQUATIONS FOR OPTIMAL PARAMETER CONSIDERATIONS

The equations provided in section IV have been analyzed here in order to understand the limits of various parameters and variables such as L_d, L_q, λ_m and their effect on the output power, reluctance torque component, optimal angle of the current vector (γ), maximum speed and hence, the constant power speed range of a six phase IPMSM.

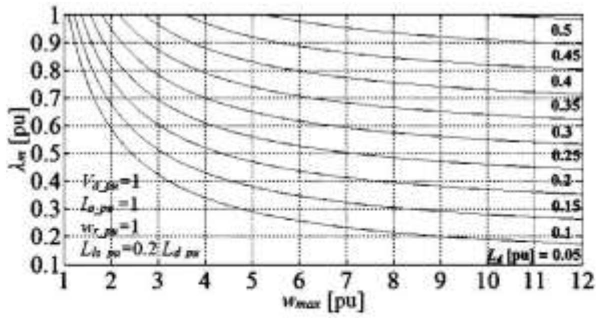
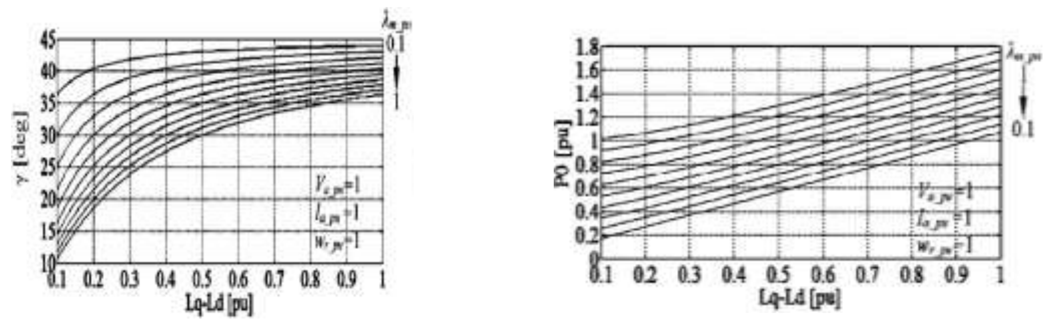


Figure 5. λ_m vs. w_{max} as a function of Changing L_d in per-unit quantities



(a) (b)

Figure 6. Gamma and output power as a function of L_q-L_d and λ_m in per-unit quantities where λ_m is varied between 0.1 and 1 pu at periodic steps of 0.1 pu. (a) γ vs. L_q-L_d. (b) Output power vs. L_q-L_d

5. CONCLUSION

The analytical d-q axes based model of a six-phase IPMSM has been formulated neglecting the mutual leakage inductance between the two numbers of stator winding phase sets. The work has presented prototype design of a six-phase interior PMSM and its parameter. The machine for which the design is considered will be used in an electrified vehicle traction application. The variation of inductances and permanent magnet flux linkage show nearly equal result with three phase IMPSM machine. As this is novel and no such machine exists in the laboratory, validation of this derived model has been performed through a more accurate finite element based formulation through ANSYS MAXWELL 2D software and results are found to be in close agreement. This model is expected to be the backbone for developing MTPA control strategy eventually on a 6-phase IPMSM drive, which will be a future scope of work.

REFERENCES

[1] E. Levi, R. Bojoi, F. Profumo, H. A. Toliyat and S. Williamson, "Multiphase induction motor drives - a technology status review," in *IET Electric Power Applications*, vol. 1, no. 4, pp. 489-516, July 2007.

[2] D. G. Dorrell, C. Y. Leong and R. A. McMahon, "Analysis and Performance Assessment of Six-Pulse Inverter-Fed Three-Phase and Six-Phase Induction Machines," in *IEEE Transactions on Industry Applications*, vol. 42, no. 6, pp. 1487-1495, Nov.-dec. 2006.

[3] T. J. E. Miller and M. I. McGilp, "Analysis of multi-phase permanent-magnet synchronous machines," *2009 International Conference on Electrical Machines and Systems*, Tokyo, 2009, pp. 1-6.

[4] E. F. Fuchs and L. T. Rosenberg, "Analysis of an Alternator with Two Displaced Stator Windings," in *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-93, no. 6, pp. 1776-1786, Nov. 1974.

[5] R. F. Schiferl and C. M. Ong, "Six Phase Synchronous Machine with AC and DC Stator Connections, Part I: Equivalent Circuit Representation and Steady-State Analysis," in *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, no. 8, pp. 2685-2693, Aug. 1983.

[6] E. Levi, "Multiphase Electric Machines for Variable-Speed Applications," in *IEEE Transactions on Industrial Electronics*, vol. 55, no. 5, pp. 1893-1909, May 2008.

[7] K. B. Kim, Y. W. Woo, "Defect Detection in Ceramic Images Using Sigma Edge Information and Contour Tracking Method," *International Journal of Electrical and Computer Engineering*, vol. 6, no.1, pp. 160-166, Feb 2016.

[8] P. C. Krause, O. Wasynczuk, and S. Sudhoff, "*Analysis of Electric Machinery and Drive Systems*," Wiley-IEEE Press, 2002.
[9] Simmons O'Kelly, "Generalised Theory of Electrical Machines"
[10] R. Schiferl and T. A. Lipo, "Power capability of salient pole permanent magnet synchronous motors in variable speed drive applications," *Conference Record of the 1988 IEEE Industry Applications Society Annual Meeting*, Pittsburgh, PA, USA, 1988, pp. 23-31 vol.1.

BIOGRAPHIES OF AUTHORS

Completed graduation in Electrical Engineering from West Bengal University of Technology in 2013, Masters of Engineering in Power Electronics and drives in 2015.His area of interest includes Electrical Machines, Power Electronics and drives. Currently working as a lecturer in Electrical Engineering in the Technical Dept. of Govt. of West Bengal.

Completed graduation in Electrical Engineering from West Bengal University of Technology in 2013, Masters of Engineering in Electrical Power System in 2015.Area of interest includes power system, Machine and machine drives and Control Applications. Current working in Power Grid Corporation of India Ltd.

Completed graduation in Electrical Engineering from West Bengal University of Technology in 2013, Masters of Engineering in Electrical Power System in 2015.Area of interest includes power system, Machine drives, Embedded Systems. Currently working in West Bengal Power Development Corporation Ltd.

Completed graduation in Electrical Engineering from West Bengal University of Technology in 2013, Masters of Technology in Energy Systems in 2015. Area of interest includes power system, Renewable Energy. Current working as a Research Scientist In University of Petroleum and Energy Studies, Dehradun, India.