International Journal of Applied Power Engineering (IJAPE)
Vol. 1, No. 2, August 2012, pp. 65~74
ISSN: 2252-8792

Transmission Loss Allocation Based on Lines Current Flow

Rahmat Azami, Ali mansori, NosratAllah Mohammad Baigi, Amin Omidian, Ehsan Mohammadian
Departement of Electrical Engineering, Science and Research Branch, Islamic Azad University, Ilam, Iran

Article Info

Article history:

Received Jun 2, 2012
Revised Jul 20, 2012
Accepted Jul 28, 2012

Keyword:

Loss allocation
Sending and receiving power
Pool market

ABSTRACT

In this paper, the transmission loss allocation problem has been studied and a new method for loss allocation in pool electricity markets has been proposed. To do that, at first using a transmission line loss equations respect to bus injected currents, the share of each bus from the mentioned transmission line losses has been determined. Then, this method has been applied to the total network transmission lines and the share of each bus from the total losses has been acquired. The proposed method is based on the main network relations and no any simplifying suppose has been used. Finally, the proposed method is studied on a typical network.

Copyright © 2012 Institute of Advanced Engineering and Science.
All rights reserved.

Corresponding Author:

Rahmat Azami,
Departement of Electrical Engineering,
Science and Research Branch, Islamic Azad University, Ilam, Iran,
Email: azami.rahmat@yahoo.com

1. INTRODUCTION

In power systems, some percentage of the transmission power is always wasted away which it is caused by various factors. The Main port of these losses is due to the flow of current in ohmic resistance in the transmission lines. In traditional power systems with uniform structure, all the attempts were made in order to minimize the network losses and in terms of costs, the overall cost of losses is added to other generation and transmission costs and constitutes the total operation cost of the network. But in deregulated power systems, any component of the system posses the separate legal character and therefore it is independent in terms of income and costs. Thus, determining their share in total network costs including the losses is unavoidable [1]. In the other hand, in deregulated power systems, regardless of losses optimizing, another serious question is posed that how much of the total cost of losses should each of the power market players pay. In the pool-based electricity market, the loss allocation helps to recognize the share of each generation or consumption unit from the total network losses so that the ISO can receive the losses costs from each of the market participants and return it to the generation companies [2].

In the markets with are based on bilateral or multilateral transactions, the losses of each transaction should be specified in the transaction content and its support source should be determined. In spite of the high importance of loss allocation to the participants, technically and economically, due to complexity, nonlinear nature and high dependence of loss function on the different variables, no comprehensive and precise method which can be practically employed has been presented so far. But due to significance of this issue, the various methods have been published in the papers which most of them have used simple assumptions. In Pro rata method [3] that is the most popular ones, the loss is allocated to each generator or load, regarding their power injection to network rather than total network power injection. In fact, this method doesn't consider the location of them or network topology. So a remote generator or load that certainly causes more power losses treats the same as other near network players.

Proportional sharing principle is based on a non provable or disprovable theorem that assumes the inflow powers are proportionally shared between the outflows power at each network bus [4]-[5]. This method uses an additional assumption that losses of each branches allocate in 50 percent to its sending and ending nodes.

Reference [6] suggested a radial equivalent network for transmission system that each generator may have an individual connection to all loads and in this way made it possible to separates system loss but total losses may not equal to real system and also is too complicated for real power systems. References [7]-[8]-[9] trace losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses attributed to generators/loads accumulate as one traverse through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss sharing formula is implicitly included.

The cooperative game theory was utilized [10] to allocate transmission costs to wheeling transactions. A method, based on circuit theory, has also been proposed to trace power from either the seller and/or the buyer's point of view [11]. In [12], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. The scheme then proposes ways in which the coupling terms among the components appearing in the line losses can be allocated to individual bilateral transactions. In [13] a process is used whereby individual bilateral transactions are gradually incremented along a given path of variation. Each bilateral transaction may elect to have its losses supplied by a separate slack generator.

In [9] starting from an ac load flow solution, the contributions of all generators to the flow in each circuit are evaluated and the same proportion is used to share circuit losses among them. The Z-bus loss allocation, use the total system loss formula and try to write it in the summation form of each bus complex current injection [14].

Finally, in [15] a loss allocation method has been introduced in bilateral markets. In order to apply the loss allocation to transactions, this method uses the branch current circuit equations. In this reference, each transaction contains a sending bus (seller) and several receiving buses (buyer). The loss allocation problem in multi-area transmission networks is studied in [16].

In this paper using a transmission line loss equations respect to bus injected currents, the share of each bus from the mentioned transmission line losses has been determined. Then, this method has been applied to the total network transmission lines and the share of each bus from the total losses has been acquired. The proposed method is based on the main network relations and no any simplifying suppose has been used. In the next sections, the proposed method is studied on a typical network.

2. PROBLEM FORMULATION

In power networks, the total loss is due to the power flows in transmission lines. In fact, the total loss is sum of the losses of all transmission lines. Suppose the power flow results, state estimation for an n-bus network with its topology are available. Subsequently using the sending and receiving power equations of a transmission line and its relation to the produced transmission losses, the share of each bus from these losses can be calculated. The connected transmission line between bus i and bus j can be considered as shown in Figure 1. According to this figure, for the mentioned line, the injected current and power can be formulated as the following [17-19]:

$$I_{ij} = I_l + I_{i\circ} = y_{ij}\left(V_i - V_j\right) + y_{i\circ}V_i \tag{1}$$

So, the equation of the injected power to the mentioned line from bus i can be written as:

$$\begin{aligned} S_{ij} &= V_i I_{ij}^* = V_i \times \left(y_{ij}\left(V_i - V_j\right) + y_{i\circ}V_i\right)^* \Rightarrow \\ &\Rightarrow y_{ij}^* V_i \left(V_i - V_j\right)^* + y_{i\circ}^* \left|V_i\right|^2 \end{aligned} \tag{2}$$

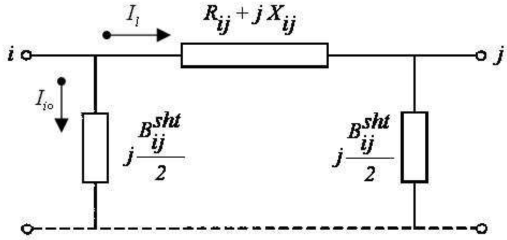

Figure 1. Schematic of a transmission line between bus i and j

In the other hand, the relations between the bus voltages with the injected currents to the buses are as the following:

$$V_i = \sum_{k=1}^{n} z_{ik} I_k$$
$$V_j = \sum_{k=1}^{n} z_{jk} I_k \tag{3}$$

By inserting (3) in (2) we have:

$$\begin{aligned} S_{ij} &= y_{ij}^* V_i \left(V_i - V_j\right)^* + y_{i\circ}{}^* \left|V_i\right|^2 = \\ & y_{ij}^* V_i \left(\sum_{k=1}^{n} z_{ik} I_k - \sum_{k=1}^{n} z_{jk} I_k\right)^* + y_{i\circ}{}^* \left|V_i\right|^2 \Rightarrow \\ & \Rightarrow S_{ij} = y_{ij}^* V_i \left(\sum_{k=1}^{n} \left(z_{ik} - z_{jk}\right) I_k\right)^* + y_{i\circ}{}^* \left|V_i\right|^2 \end{aligned} \tag{4}$$

Where S_{ij} is the injected power from the bus i to the mentioned line, y_{ij} is the series admittance of line, z_{ik} is the element of row i and column k of the network impedance matrix, z_{jk} is the element of row j and column k of the network impedance matrix, $y_{i\circ}$ is the shunt admittance of line, V_i is voltage of bus i and I_k is the injected current to the bus k.

Similarly, the injected current and power from bus j to the mentioned line can be calculated as the following:

$$I_{ji} = -I_l + I_{j\circ} = y_{ij}\left(V_j - V_i\right) + y_{j\circ} V_j \tag{5}$$

$$\begin{aligned} S_{ji} &= V_j I_{ji}^* = V_j \times \left(y_{ji}\left(V_j - V_i\right) + y_{j\circ} V_j\right)^* \Rightarrow \\ & \Rightarrow y_{ij}^* V_j \left(V_j - V_i\right)^* + y_{j\circ}{}^* \left|V_j\right|^2 \end{aligned} \tag{6}$$

Now by inserting (3) in (6) we have:

$$S_{Lij} = S_{ij} + S_{ji} \tag{7}$$

$$\begin{aligned} S_{ji} &= y_{ij}^* V_j \left(V_j - V_i\right)^* + y_{j\circ}{}^* \left|V_j\right|^2 = \\ & y_{ij}^* V_j \left(\sum_{k=1}^{n} z_{jk} I_k - \sum_{k=1}^{n} z_{ik} I_k\right)^* + y_{j\circ}{}^* \left|V_j\right|^2 \Rightarrow \\ & \Rightarrow S_{ji} = y_{ij}^* V_j \left(\sum_{k=1}^{n} \left(z_{jk} - z_{ik}\right) I_k\right)^* + y_{j\circ}{}^* \left|V_j\right|^2 \end{aligned} \tag{8}$$

Then by inserting (4) and (7) in (8), equation (9) can be deduced:

$$\begin{aligned} S_{Lij} &= S_{ij} + S_{ji} = y_{ij}^* V_i \left(\sum_{k=1}^{n} \left(z_{ik} - z_{jk}\right) I_k\right)^* + \\ & y_{i\circ}{}^* \left|V_i\right|^2 + y_{ij}^* V_j \left(\sum_{k=1}^{n} \left(z_{jk} - z_{ik}\right) I_k\right)^* + y_{j\circ}{}^* \left|V_j\right|^2 \Rightarrow \\ & \Rightarrow S_{Lij} = y_{i\circ}{}^* \left|V_i\right|^2 + y_{j\circ}{}^* \left|V_j\right|^2 \\ & + y_{ij}^* \left(\sum_{k=1}^{n} \left(z_{ik} - z_{jk}\right) I_k\right)^* \left(V_i - V_j\right) \end{aligned} \tag{9}$$

Using the above equation, the share of bus k from the losses of line i-j can be calculated as:

$$S_{Lij}^{k}=\left(y_{i\circ}{}^{*}|V_i|^2+y_{j\circ}{}^{*}|V_j|^2\right)\times\frac{|(z_{ik}-z_{jk})I_k|}{\sum_{k=1}^{n}|(z_{ik}-z_{jk})I_k|}$$
$$+y_{ij}^{*}((z_{ik}-z_{jk})I_k)^{*}(V_i-V_j) \tag{10}$$

The losses equation (9) can be rewritten with respect to the current relations as the following:

$$S_{Lij}=y_{i\circ}{}^{*}|V_i|^2+y_{j\circ}{}^{*}|V_j|^2+y_{ij}^{*}\left(\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right)^{*}(V_i-V_j)$$
$$=y_{i\circ}{}^{*}|V_i|^2+y_{j\circ}{}^{*}|V_j|^2+y_{ij}^{*}\left(\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right)\left(\sum_{k=1}^{n}(z_{ik}-z_{jk})^{*}I_k^{*}\right) \tag{11}$$
$$\Rightarrow S_{Lij}=y_{i\circ}{}^{*}|V_i|^2+y_{j\circ}{}^{*}|V_j|^2+y_{ij}^{*}\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2$$

The real part of the losses in line i-j is as the below:

$$P_{Lij}=\Re\{S_{Lij}\}=\Re\left\{y_{i\circ}{}^{*}|V_i|^2+y_{j\circ}{}^{*}|V_j|^2+y_{ij}^{*}\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\} \tag{12}$$

According to the figure (1), the shunt admittance only has imaginary value so the firs and second terms in equation (12) are zero. Then, the real part of losses is as the following:

$$y_{i\circ}=y_{j\circ}=j\frac{B_{ij}^{sht}}{2}\Rightarrow\Re\left\{y_{i\circ}{}^{*}|V_i|^2\right\}+\Re\left\{y_{j\circ}{}^{*}|V_j|^2\right\}=0\Rightarrow$$
$$\Rightarrow P_{Lij}=\Re\left\{y_{ij}^{*}\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\} \tag{13}$$

In the other hand, the admittance between buses i and j with respect to the elements of the network admittance matrix can be calculated as:

$$y_{ij}^{*}=\text{-}Y_{ij}^{*} \tag{14}$$

By inserting (14) in (13) we have:

$$P_{Lij}=\Re\left\{-Y_{ij}^{*}\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\} \tag{15}$$

Where Y_{ij} is the element of row i and column j from network admittance matrix (Y). The Y matrix can be written as the below:

$$Y=G+jB=\begin{bmatrix}G_{11}&G_{12}&\cdots&G_{1n}\\G_{21}&G_{22}&\cdots&G_{2n}\\\vdots&\vdots&\vdots&\vdots\\G_{n1}&G_{n2}&\cdots&G_{nn}\end{bmatrix}+j\begin{bmatrix}B_{11}&B_{12}&\cdots&B_{1n}\\B_{21}&B_{22}&\cdots&B_{2n}\\\vdots&\vdots&\vdots&\vdots\\B_{n1}&B_{n2}&\cdots&B_{nn}\end{bmatrix} \tag{16}$$

By inserting (16) in (15) we have:

$$P_{Lij}=\Re\left\{-(G_{ij}+jB_{ij})^{*}\times\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\}\Rightarrow$$
$$\Rightarrow P_{Lij}=\Re\left\{-G_{ij}\times\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\} \tag{17}$$
$$+\Re\left\{jB_{ij}\times\left|\sum_{k=1}^{n}(z_{ik}-z_{jk})I_k\right|^2\right\}$$

But the second term has always the zero value so, (17) can be reduced to equation (18):

$$P_{Lij} = \left\{ -G_{ij} \times \left| \sum_{k=1}^{n} \left(z_{ik} - z_{jk} \right) I_k \right|^2 \right\} \tag{18}$$

By extension the equation (18) we have:

$$P_{Lij} = \left\{ -G_{ij} \times \left| \begin{matrix} (z_{i1} - z_{j1}) I_1 + (z_{i2} - z_{j2}) I_2 \\ + (z_{i3} - z_{j3}) I_3 + \cdots + (z_{in} - z_{jn}) I_n \end{matrix} \right|^2 \right\} \tag{19}$$

Now the share of the injected current of bus k in losses of line i-j is as the following:

$$\begin{aligned} P_{Lij}^k = & -G_{ij} \times (\left|(z_{ik} - z_{jk}) I_k\right|^2 + \\ & \sum_{m=1,\neq k}^{n} \left\{ \begin{matrix} 2\times\Re((z_{ik}-z_{jk})I_k)\times\Re((z_{im}-z_{jm})I_m) \\ \times\dfrac{\left|\Re((z_{ik}-z_{jk})I_k)\right|}{\left|\Re((z_{ik}-z_{jk})I_k)\right| + \left|\Re((z_{im}-z_{jm})I_m)\right|} + \\ \Im((z_{im}-z_{jm})I_m) \\ 2\times\Im((z_{ik}-z_{jk})I_k)\times\displaystyle\sum_{m=1,\neq k}^{n}\times\dfrac{\left|\Im((z_{ik}-z_{jk})I_k)\right|}{\left|\Im((z_{ik}-z_{jk})I_k)\right| + \left|\Im((z_{im}-z_{jm})I_m)\right|} \end{matrix} \right\} \end{aligned} \tag{20}$$

Then, the share of bus k from the total losses of network can be calculated as the below:

$$\begin{aligned} & Ploss^k = \sum_{i=1}^{n}\sum_{j=1}^{n} 0.5\times P_{Lij}^k \Rightarrow \\ & Ploss^k = \sum_{i=1}^{n}\sum_{j=1}^{n} -0.5\times G_{ij}\times \\ & \left\{ \begin{matrix} \left|(z_{ik}-z_{jk})I_k\right|^2 + 2\times\Re((z_{ik}-z_{jk})I_k)\times \\ \displaystyle\sum_{m=1,\neq k}^{n} \Re((z_{im}-z_{jm})I_m)\times\dfrac{\left|\Re((z_{ik}-z_{jk})I_k)\right|}{\left|\Re((z_{ik}-z_{jk})I_k)\right| + \left|\Re((z_{im}-z_{jm})I_m)\right|} + \\ 2\times\Im((z_{ik}-z_{jk})I_k)\times\displaystyle\sum_{m=1,\neq k}^{n}\Im((z_{im}-z_{jm})I_m) \\ \times\dfrac{\left|\Im((z_{ik}-z_{jk})I_k)\right|}{\left|\Im((z_{ik}-z_{jk})I_k)\right| + \left|\Im((z_{im}-z_{jm})I_m)\right|} \end{matrix} \right\} \end{aligned} \tag{21}$$

3. NUMERICAL RESULTS

In order to show the results of the proposed method and compare it with other methods, the IEEE 14-bus system has been chosen. As can be observed in Figure 2, the IEEE 14-bus system has 5 voltage controlled buses and 2 generators. The bus no.1 is chosen as the slack bus.

The load flow results of Table 1 show that 13.4 MW of losses are supplied by bus no.1 which should be divided between the market customers.

Table 2 shows the results of the proposed method in comparison to the other methods. Considering this equation and the values of Table 1, it is seen that the buses no. 5, 8, 11 have the minimum rate of loss changes in response to current injection which shows that the corresponding buses act in the direction of loss reduction.

The proposed method, similar to the impedance matrix method, emphasizes on the location of buses and network topology. According to Tab. 3, bus 1 that provides about 72 percent of the total generation always has the highest contribution to the loss allocation in the all methods. Also, bus 3 which comprises 11.5 percent of system load, after bus 1, receives the highest loss allocation by all methods. Bus 2, because of the appropriate location in the network has the least loss allocation value.

In order to analyze the effect of distributed generation and consumption, a 100 MW generator is added to the bus 8.

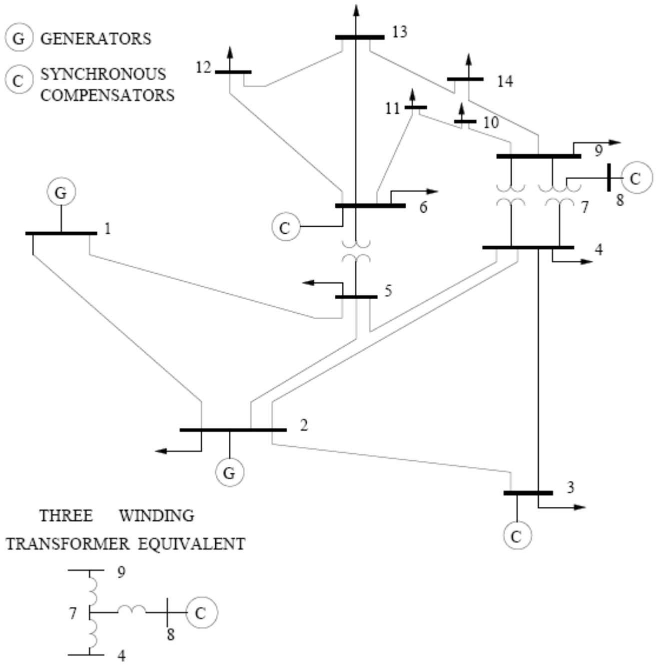

Figure 2. 14-bus IEEE test system

Table 1. Results of a normal load flow analysis

Bus No	voltage	angle	Pg	Qg	Pd	Qd
1	1.060	0.000	232.39	16.5-	0.00	0.00
2	1.045	4.983-	40.00	30.86	21.7	12.7
3	1.010	12.72-	0.000	6.000	94.2	19.0
4	1.018	10.31-	0.000	-3.90	47.8	-3.90
5	1.020	-8.774	0.000	-1.60	7.60	1.60
6	1.070	14.22-	0.000	5.000	11.2	7.50
7	1.062	-13.36	0.000	0.000	0.00	0.00
8	1.090	-13.36	0.000	17.62	0.00	0.00
9	1.056	14.93-	0.000	-16.6	29.5	16.6
10	1.051	15.09-	0.000	-5.80	9.00	5.80
11	1.057	14.79-	0.000	-1.80	3.50	1.80
12	1.055	15.07-	0.000	-1.60	6.10	1.60
13	1.050	15.15-	0.000	-5.80	13.5	5.80
14	1.036	16.03-	0.000	-5.00	14.9	5.00
sum			272.39	82.44	259.0	73.5

Table 2. Results of loss allocation from different methods and proposed method

Bus Number	Z-bus method	ITL method	Pro-rata method	Proposed method
1	7.800	6.14	6.46	9.68
2	0.155	0.96	0.50	0.04
3	2.698	2.92	2.62	1.55
4	0.9056	1.26	1.36	0.57
5	0.0903	0.18	0.22	0.01
6	0.6783	0.32	0.32	0.39
7	0.000	0.000	0.000	0.000
8	0.0258	0.000	0.000	0.09
9	0.4484	0.68	0.82	0.20
10	0.1690	0.20	0.24	0.08
11	0.0620	0.08	0.10	0.01
12	0.1385	0.18	0.16	0.07
13	0.3412	0.32	0.38	0.26
14	0.4689	0.32	0.42	0.44
sum	13.39	13.39	13.39	13.39

Table 3. The results of proposed method beside the other methods with adding a 100 MW generator to bus 8of the IEEE 14-bus network

Bus Number	Z-bus method	ITL method	Pro-rata method	Proposed method
1	2.32	1.80	1.60	2.5838
2	0.08	0.56	0.24	0.1588
3	2.48	1.58	1.20	2.3168
4	0.26	0.50	0.62	0.5281
5	0.02	0.08	0.10	0.0331
6	0.46	0.20	0.14	0.1418
7	0.00	0.00	0.00	0.00
8	-0.18	0.88	1.28	-0.8595
9	0.06	0.14	0.38	0.2956
10	0.06	0.06	0.12	0.1055
11	0.02	0.04	0.04	0.0221
12	0.10	0.10	0.08	0.0664
13	0.22	0.16	0.18	0.2605
14	0.30	0.10	0.20	0.3850
Total Sum	6.03	6.03	6.03	6.03

Table. 3 shows the main variation of transmission system losses, which has been led to 50 percent decrease in total network losses. Therefore, the allocated loss to the buses has changed and share of bus 1 decreased from 62 percent in previous state to 42 percent in this condition. But, due to the high distance of bus 3 from generation center, its contribution to the allocated loss has no main variations.

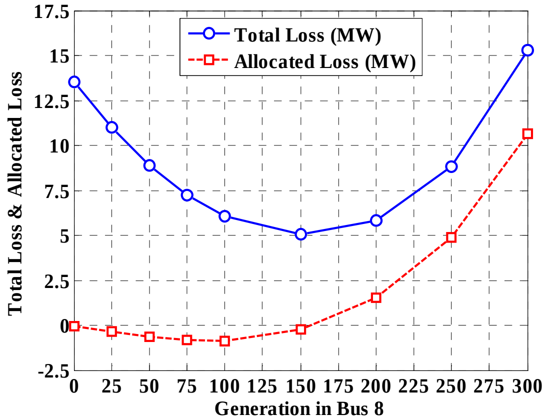

Figure 3. the total network losses and the allocated loss to bus 8 by changing its generation from zero to 300 MW

By varying the generation of bus 8 from zero to 300 MW, as illustrated in Figure 3, the network losses firstly decrease and then increase. Also, the allocated loss, that firstly decreases and then increases, shows the proposed method has considered the network topology and the injected currents to the network buses.

4. CONCLUSION

In this paper, a fair method has been proposed to allocate the transmission losses in a power system using its circuit equations and simplifying them. This method divided the losses between the players of a pool-based market using the network impedance matrix and the partial derivatives of the active power losses respect to bus currents coefficients. The method is based on load flow and the following principles:

1- Incorporates the main equations of the power system in conjunction with the network impedance matrix and the vectors of the injected currents of the buses.

2- Uses the partial derivatives of the active power losses respect to bus currents coefficients for fair allocation of losses between the network customers.

3- It is a simple and easily understandable method.

The proposed method in this paper doesn’t consider any bus or buses compensating the network total losses. It is actually independent of the slack bus and divides the losses between the market players considering the penetration percent of them in the network. The method separates the self and mutual losses and is therefore applicable in other forms of the power system such as multi-transaction contract markets. It can actually be used to compensate the losses by the buses, themselves. Finally, the proposed method has been tested on the IEEE 14-bus system and fair results have been achieved in comparison to the other methods.

REFERENCES

[1] M. Ilic, F. Galiana, and L. Fink,” Power Systems Restructuring: Engineering and Economics”, Norwell, MA: Kluwer, 1998.

[2] G. B. Sheblé,“ Computational Auction Mechanisms for Restructured Power Industry Operation”, Norwell, MA: Kluwer, 1999.

[3] A.J. Conejo and et al.,“ Transmission Loss Allocation: A Comparison of Different Practical Algorithms”, IEEE Transaction on Power Systems, vol. 17, no. 3, August 2002

[4] J. W. Bialek, “Tracing the flow of electricity”, IEE Proc. Gener.,Trans., and Distrib, vol. 143, pp. 313-320, 1996.

[5] D. Kirschen and G. Strbac,“ Tracing active and reactive power between generators and loads using real and imaginary currents”, IEEE Trans. Power System, vol. 14, pp. 1312-1319, Nov. 1999.

[6] N. S. Rau,“ Radial equivalents to map networks to market formats” IEEE Trans. Power System, vol. 16, no. 4, pp. 856-861, Nov. 2001.

[7] J.W. Bialek,“ Tracing the flow of electricity" IEE Proc., Gener. Transm. Distrib, vol 143, pp. 313–320, 1996.

[8] (6) C. N. Macqueen, and M.R. Irving, "An algorithm for the allocation of distribution system demand and energy losses" IEEE Trans. Power Syst, 1996, 11, (1), pp. 147–156

[9] D. Kirschen, R. Allan, and G. Strbac, "Contributions of individual generators to loads and flows" IEEE Trans. Power Syst, 1997, 12, (1), pp. 52–60

[10] Y. Tsukamoto, and I. Iyoda, " Allocation of fixed transmission cost to wheeling transactions by cooperative game theory" IEEE Trans. Power Syst, 1996, 11, (2), pp. 620-627

[11] Y.C. Chang, and C. N. Lu, "An electricity tracing method with application to power loss allocation" Int. J. Elec. Power Energy Sys, 2000. 23, (1), pp. 13-17

[12] A. Gomez Exposito, J. M. Riquelme Santos, T. Gonzalez Garcia, and E. A. Ruiz Velasco, “Fair allocation of transmission power losses,” IEEE Trans. on Power Systems, vol. 15, no. 1, pp. 184–188, Feb. 2000.

[13] F. D. Galiana and M. Phelan, “Allocation of transmission losses to bilateral contracts in a competitive environment,” IEEE Trans. on Power Systems, vol. 15, no. 1, pp. 143–150, Feb. 2000.

[14] A. J. Conejo and et. al, "Z-bus loss allocation" IEEE Trans. Power System, vol. 16, pp. 105-110, Feb. 2001.

[15] C. Adsoongnoen, W. Ongsakul, C. Maurer, H. Haubrich " Transmission pricing using the exact power and loss allocation method for bilateral contracts in a deregulated electricity supply industry" Euro. Trans. Electr. Power 2007; 17:240–254 DOI: 10.1002/etep.131

[16] A. Kazemi, S. Jadid, H. Andami “A circuit based method for multiarea transmission networks loss allocation" Euro. Trans. Electr. Power 2008; 18:753–766 DOI: 10.1002/etep.225

[17] R. Aazami, M. S. Javadi, H. Monsef, H. Lessani“A Methodology for Active Power Transmission Loss Allocation in Power Markets” Int. Rev. Elec. Eng (IREE), Vol. 4, No. 4, pp. 577–582, July-August 2009.

[18] Akash Davda, Dr. M.D.Desai, Dr. B.R.Parekh,” Assessment of Reduction in Losses by Distributed Generation Penetration in a Radial Network: a Case Study” Vol. 4 N. 1, pp. 120-124, February 2011.

[19] B. Noshad, M. S. Javadi, M. Joorabian, M. Goodarzi, A. Javadinasab, “Reactive Power Pricing Emphasizing on Voltage Stability”, Vol. 3 N. 6, pp. 1501-1508, December 2010.