International Journal of Applied Power Engineering (IJAPE)
Vol. 2, No. 3, December 2013, pp. 141~148
ISSN: 2252-8792

Probabilistic Q-Margin Calculations Considering Dependency of Uncertain Load and Wind Generation

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Article Info

Article history:

Received Mar 15, 2013
Revised Aug 27, 2013
Accepted Sep 5, 2013

Keyword:

Discrete joint distribution
Reactive power (Q) margin
Voltage stability
Wind power

ABSTRACT

This paper presents a novel probabilistic approach for computation of the reactive power margin (or Q-margin) of a power system with large-scale uncertain wind generation. Conventionally, Q-margin has been used as an index for indicating the system voltage stability level on system operation and planning. The conventional Q-margin method needs to be modified to fully accommodate uncertainties due to wind generation. This paper proposes a new Q-margin computation method using Q-matrix and the expected Q-margin (EQM). Q-matrix is a generic uncertainty matrix representing a discrete joint distribution of load and wind generation and the EQM, calculated from the Q-matrix, is a specific probabilistic variable that supersedes the conventional Q-margin. The proposed method is verified with the IEEE 39-bus test system including wind generation.

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1. INTRODUCTION

One of the serious concerns about integration of large-scale wind power into the power system is the impact of generation uncertainty on voltage stability [1]. The theory of voltage stability suggests that more reactive power support is needed if the transmission lines are heavily loaded. Then if reactive power is not properly managed, the corresponding bus voltages at the receiving ends will slip to the unstable region, and potentially cause local voltage instability, or at worst, a widespread system voltage collapse. Large-scale wind generation not only affects the voltages at the receiving ends, but its intermittent nature introduces a great deal of uncertainty, which further intensifies its negative impact on voltage [2],[3]. Increasing occurrence of voltage problems has already been experienced in several control areas with fast growth of wind generation. This places an urgent request for new methodologies for assessment of voltage stability.

One of the critical topics in voltage stability analysis is assessment of reactive power adequacy. Adequate reactive power ensures the static voltage stability. This is an essential assumption for many other reliability/security studies, including transient stability analysis. Two important indices are often used: active power margin (P-margin) and reactive power margin (Q-margin) [4]. Both indices measure how close the current voltage level is to the instability region. They are also used to find where and how much reactive power needs to be compensated. The current methodologies of Q-margin estimation are based on deterministic approaches [5]-[8]. These deterministic approaches only consider certain worst-case scenarios associated with the load. Although these approaches work fine if the load is the only major uncertainty, they become inadequate if significant resource-side uncertainties of the power balancing equation are introduced.

This paper presents a novel probabilistic approach for Q-margin computation of a power system with uncertain wind generation. The proposed approach takes into account the uncertainties of the load and wind generation through a dependence matrix. Along with the conventional deterministic methods, this new approach can be used to find the distribution of Q-margin. The Q-margin distribution provides essential information to support operational decisions under uncertainty. An illustrative example is presented to show a computation of the expected Q-margin (EQM) based on its distribution in the IEEE 39-bus testing system.

The remainder of this paper is organized as follows: Section 2 provides a brief overview of the conventional Q-margin and QV analysis; Section 3 describes the uncertainty matrix model for the load and wind generation; Section 4 presents how to apply the proposed method to a sample system; and Section 5 discusses the simulation results; Section 6 is the conclusion of this paper.

2. Q-MARGINAND QV ANALYSIS

Assessment of reactive power adequacy in system voltage stability analysis can be explained by the QV curve [7]. A QV curve usually is constructed numerically based on a series of load-flow analysis. A typical QV curve has a convex contour as shown in Figure 1.

QV curve describes the variation of bus voltages with respect to reactive power injections and consumptions. In Figure 1, the lowest level of the curve is the maximum Mvar load, at which the derivative dQ/dV equals to zero. Additional consumption of reactive power beyond this critical level will drive the bus voltage to the left side of the dashed vertical line, i.e., the unstable region. The reactive power and the voltage associated with the critical level can be used to determine the minimum reactive power needed for voltage stability.

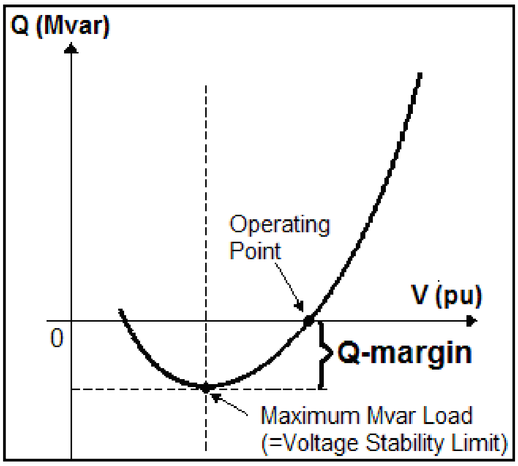


Figure 1. A Typical QV curve

Once the voltage stability limit is known, the QV curve can be used to determine how much reactive power is needed from such voltage compensation devices as shunt capacitors, static VAR compensators (SVC), or Static Synchronous Compensators (STATCOM), to improve the security cushion for voltage collapse risk.

How far the current operation state is to the critical voltage stability limit usually is represented by the Q-margin. Q-margin is the difference between the reactive power of the voltage stability limit and that of the current operating point.

Q-margin of any given scenario can be obtained through load flow analysis with a fictitious synchronous condenser (SC) attached to the bus of interest [8]. Each point on the QV curve can be found based on the amount of reactive power needed from SC for maintaining the given voltage setting point.

3. A MATRIX MODEL OF UNCERTAINTIES OF LOAD AND WIND GENERATION

3.1. Two Key Modeling Issues

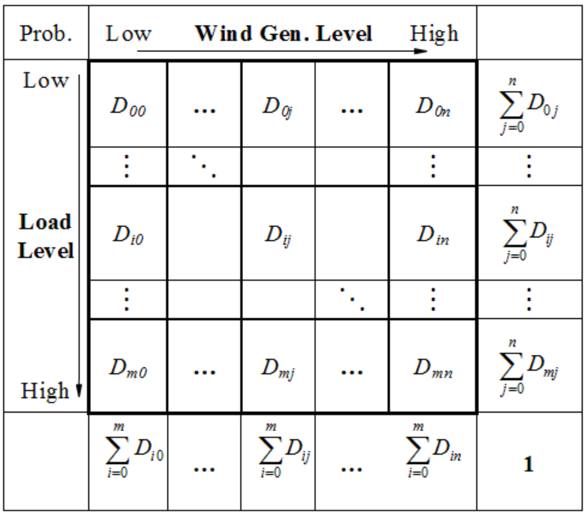
There are two essential issues in modeling the uncertainties of the load and wind generation: first, how to capture the coincidence of the uncertain load and wind generation in modeling; second, how to incorporate the modeled uncertainties into Q-margin estimation.

The key to uncertainty modeling depends on if the “inter-correlation” between the load and the wind generation can be appropriately incorporated. For example on, a strong negative dependence between the load and wind generation of a large wind farm at the Midwest ISO has been reported [9]. Although such dependence profiles could be different at the different locations and time horizons, a well-designed uncertainty model should be able to identify such dependences, if they exist. Incorporation of the inter-correlation will significantly improve the effectiveness of the uncertainty model [10].

Another major issue of uncertainty modeling is to incorporate those uncertainties in current practical methods. It is desirable to have a flexible model that can be easily incorporated into the current methods. In addition, such models should be able to provide insights on the impact of the uncertainties. These factors are considered in the proposed method.

3.2. Matrix Structure of Uncertainties

Three essential issues in uncertainty modeling can be well addressed with a matrix structure as shown Figure 2.



Prob.	Low		**Wind Gen. Level**		High	
Low	D_{00}	…	D_{0j}	…	D_{0n}	$\sum_{j=0}^{n} D_{0j}$
	⋮	⋱			⋮	⋮
Load Level	D_{i0}		D_{ij}		D_{in}	$\sum_{j=0}^{n} D_{ij}$
	⋮			⋱	⋮	⋮
High	D_{m0}	…	D_{mj}	…	D_{mn}	$\sum_{j=0}^{n} D_{mj}$
	$\sum_{i=0}^{m} D_{i0}$	…	$\sum_{i=0}^{m} D_{ij}$	…	$\sum_{i=0}^{m} D_{in}$	**1**

Figure 2. ATypical structure of an uncertainty matrix

The matrix shown in Figure 2 contains the following information: first, each entry represents a scenario for given load and wind generation levels defined by D_{ij}; second, the number in each entry (D_{ij}) represents the discrete joint probability, or weights, of the corresponding scenario; third, the coincidence between the load and wind generation is described by the distribution of weights of all entries.

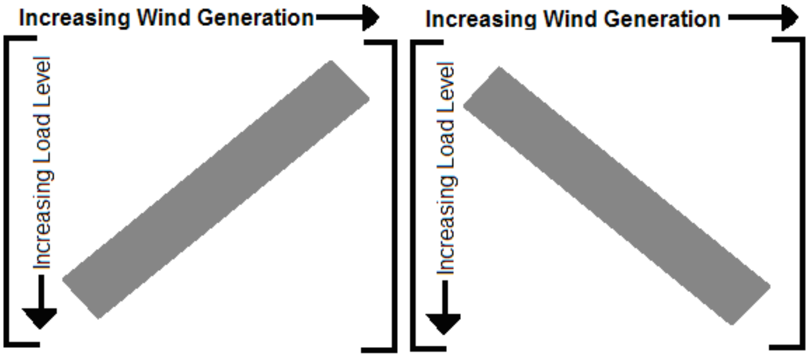


Figure 3. Matrices with (a) negative dependenceand (b) positive dependence

For example, the negative dependence between the load and wind generation will have the mass of the weights distributed around the diagonal position shown in Figure3(a). While a positive dependence between both variables will distribute the mass of weights around the diagonal direction in Figure3(b). If there is no strong inter-correlation between both variables, the mass will be evenly dispersed over the matrix.

The matrix model as shown in Figure 2 reflects the finite discretizaton of the joint distribution of two random numbers, i.e., the load and wind generation. The probability of each entry or case, D_{ij}, can be obtained based on real observations. This probability D_{ij} can be determined statistically:

$$D_{ij} = \frac{\text{Number of Occurences of Case (i, j)}}{\text{Total Observations}} \tag{1}$$

This uncertainty matrix can be formulated once all the probabilities (D_{ij}, for all i, j) are found. The columns in the matrix represent different levels of wind generation output and the rows represent different load levels. Since each entry has the likelihood of a given load and wind generation combination, it is a non-negative real number between 0 and 1. All coefficients in the uncertainty matrix are probability densities, the following condition must hold:

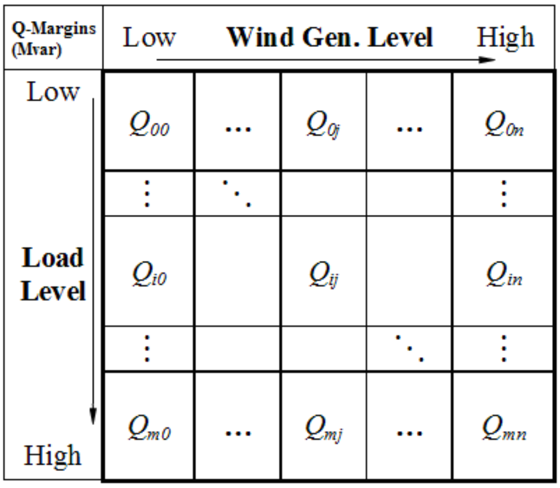
$$\sum_{i=0}^{m}\sum_{j=0}^{n} D_{ij} = 1 \tag{2}$$

where, D_{ij} is the coefficient in column i row j in the matrix.

The condition of (2) can be satisfied with the appropriate partition of the observed data.

3.3. Distribution of Q-Margin

For a given scenario defined in the uncertainty matrix, e.g., D_{ij}, the conventional methods can be used to compute the Q-margins. Once the Q-margins of all scenarios are found, a Q-margin matrix can be obtained, which is the same dimension of the uncertainty matrix and contains all the computed Q-margin values under the scenario, as shown in Figure 4.



Q-Margins (Mvar)	Low		Wind Gen. Level		High
Low	Q_{00}	…	Q_{0j}	…	Q_{0n}
	⋮	⋱			⋮
Load Level	Q_{i0}		Q_{ij}		Q_{in}
	⋮			⋱	⋮
High	Q_{m0}	…	Q_{mj}	…	Q_{mn}

Figure 4. Q-Margin matrix

Q-margin values in the matrix form represent Q-margin distribution and the Q-margin matrix can be used to support the operation decision under uncertainty. More specifically, with this distribution, the risk in Q-margin thus can be quantified due to integration of wind generation. For example, the expected Q-margin (EQM) can be found with the following equation,

$$EQM = \sum_{j=0}^{n}\sum_{i=0}^{m}\left(D_{ij} \times Q_{ij}\right) \tag{3}$$

whereQ_{ij} is i-th row and j-th column entry in the Q-margin matrix respectively. Other risk measures, such as variance of Q-margin (VQM) can also be readily obtained,

$$VQM = \sum_{j=0}^{n}\sum_{i=0}^{m}\left(D_{ij} \times \left(Q_{ij} - EQM\right)^{2}\right) \tag{4}$$

There are various advantages in adopting the matrix structure for computation of Q-margin distribution. First, the matrix model is able to incorporate the coincidence information, if there is any. Second, the matrix model can be easily implemented for any bus locations. Third, the matrix model can be used for analysis of different time horizons such as hourly, daily, weekly and monthly, etc. Fourth, the estimation of the probabilities/weights is easily implemented. Fifth, it can be incorporated into the current methods for estimation of reliability indices. In addition, during the process of construction, the uncertainty matrix may offer valuable insights on voltage stability. Many other methods for uncertainty analysis, e.g., Monte Carlo simulation, couldn’t provide much information during the simulation process.

4. CASE STUDY

4.1. System Configuration

This section presents a Q-margin analysis with the proposed method for Q-margin analysis for the IEEE 39-bus system as described in [11]. There are six wind farms inside the wind generation area of the upper right corner as shown in Figure 5. The wind farms are represented by circled W’s. In this testing power system, a load zone is located in the middle and the surrounding represents the conventional generation area.

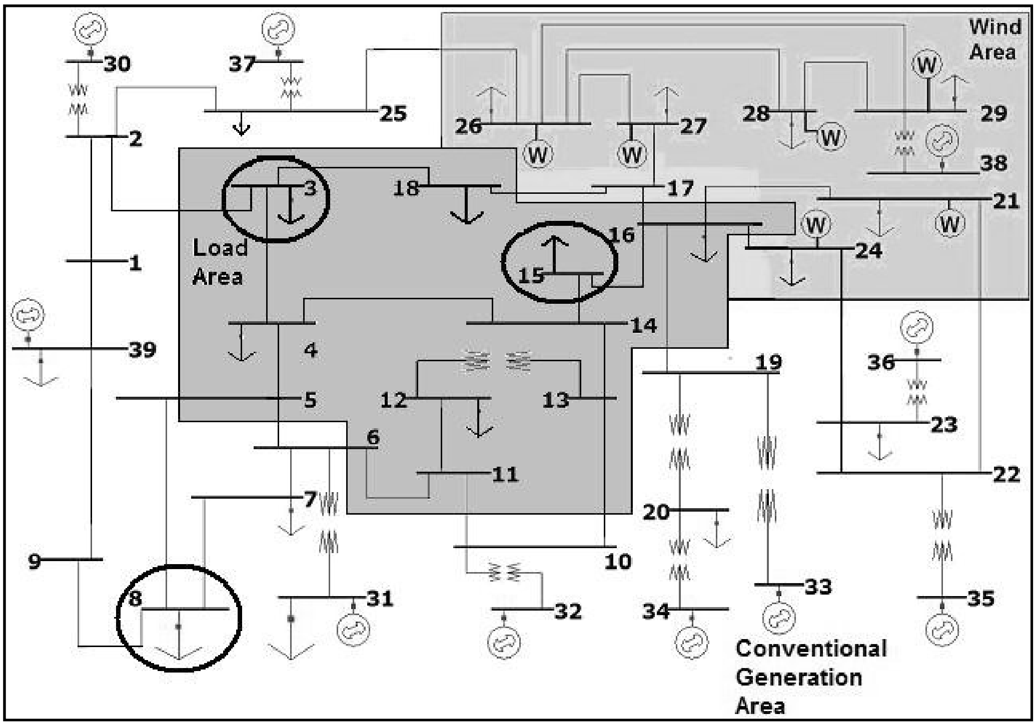


Figure 5. IEEE 39-bus Testing System with Six Wind Farms

To simplify the illustration, ten levels of load and ten levels of wind generation are presented as defined in Table II. The range of change of wind generation is 100% from zero MW to the maximum total capacity of 684MW. Without losing generality, the load changes are assumed proportionally with a range from -5% to 5%, the difference between peak load and base load is about 613MW. The total capacity of conventional generation is 6187MW. Therefore, the capacity penetration ratio (PR) of wind generation in this study is around 10%, calculated based on (5):

$$\mathrm{PR}(\%) = \frac{\mathrm{Wind\ Gen.\ Capacity}}{\mathrm{Conventional\ Gen.\ Capacity} + \mathrm{Wind\ Gen.\ Capacity}} = \frac{684\mathrm{MW}}{6187\mathrm{MW} + 684\mathrm{MW}} \times 100 \approx 10\% \qquad (5)$$

Table I shows the defined case study scenarios with case indices from 00 to 99.

Table I. Case Number Assignment

Case No.		Low	Wind Farm Generations (MW)								High
		0	*76*	*152*	*228*	*304*	*380*	*456*	*532*	*608*	*684*
Low	*5537.2*	00	01	02	03	04	05	06	07	08	09
	5587.7	10	11	12	13	14	15	16	17	18	19
	5655.8	20	21	22	23	24	25	26	27	28	29
	5723.9	30	31	32	33	34	35	36	37	38	39
Loads	*5792.1*	40	41	42	43	44	45	46	47	48	49
(MW)	*5860.2*	50	51	52	53	54	55	56	57	58	59
	5928.4	60	61	62	63	64	65	66	67	68	69
	5996.5	70	71	72	73	74	75	76	77	78	79
	6064.7	80	81	82	83	84	85	86	87	88	89
High	*6150.5*	90	91	92	93	94	95	96	97	98	99

To identify the buses which may have voltage problems, a set of contingency analysis has been performed under N-1 criteria for the 100 cases defined in Table I. For each case study scenario defined, the occurrences of significant voltage drops (below 0.9p.u.) are counted for each bus. Without losing generality, we will consider those with high frequency of voltage drops as "weak" buses.

Through the contingency analysis, bus 3, 8 and 15, are selected as the candidates for Q-margin computation because there are high occurrences of voltage problems at three buses. For only bus 8,the simulation results will be presented in Section 5 as the representative of three selected buses becausethe similar results are shown for the other selected buses.

4.2. Uncertainty Matrix

The uncertainty matrix of load and wind generation can be constructed with (1) and (2) based on historical observations. In this example, three types of matrices are arbitrarily given for illustration purposes: positive dependence matrix, negative dependence matrix and independence matrix. Table II and Table III show a positively dependent case anda negatively dependent case between the load and wind generation, respectively. For the independence matrix used in this example, all entries in the matrix are set equal to 0.01.

Table II. Postive Dependence

Positive		Low	Wind Farm Generations (MW)								High
		0	*76*	*152*	*228*	*304*	*380*	*456*	*532*	*608*	*684*
Low	*5537.2*	0.1	0	0	0	0	0	0	0	0	0
	5587.7	0	0.1	0	0	0	0	0	0	0	0
	5655.8	0	0	0.1	0	0	0	0	0	0	0
	5723.9	0	0	0	0.1	0	0	0	0	0	0
Loads	*5792.1*	0	0	0	0	0.1	0	0	0	0	0
(MW)	*5860.2*	0	0	0	0	0	0.1	0	0	0	0
	5928.4	0	0	0	0	0	0	0.1	0	0	0
	5996.5	0	0	0	0	0	0	0	0.1	0	0
	6064.7	0	0	0	0	0	0	0	0	0.1	0
High	*6150.5*	0	0	0	0	0	0	0	0	0	0.1

Table III. Negative Dependence

Positive		Low	Wind Farm Generations (MW)								High
		0	*76*	*152*	*228*	*304*	*380*	*456*	*532*	*608*	*684*
Low	*5537.2*	0	0	0	0	0	0	0	0	0	0.1
	5587.7	0	0	0	0	0	0	0	0	0.1	0
	5655.8	0	0	0	0	0	0	0	0.1	0	0
	5723.9	0	0	0	0	0	0	0.1	0	0	0
Loads	*5792.1*	0	0	0	0	0	0.1	0	0	0	0
(MW)	*5860.2*	0	0	0	0	0.1	0	0	0	0	0
	5928.4	0	0	0	0.1	0	0	0	0	0	0
	5996.5	0	0	0.1	0	0	0	0	0	0	0
	6064.7	0	0.1	0	0	0	0	0	0	0	0
High	*6150.5*	0.1	0	0	0	0	0	0	0	0	0

5. CASE STUDY

5.1. Q-margin Matrix

The Q-margin matrix for bus 8 consists of Q-margins which are computed inall case scenarios. For example, for case 24 as defined in Table I, the total system load is 5655.8MW and the wind generation level is 304MW. In case 24, Q-margin for bus 8 is 1321.2MW. Analogously Q-margins for other cases are found in Table IV.

Table IV. Q-Margin Matirx for Bus 8

Q-margins Bus 8		Low			**Wind Farm Generations (MW)**				High		
		0	***76***	***152***	***228***	***304***	***380***	***456***	***532***	***608***	***684***
Low	***5537.2***	1390.0	1368.2	1341.7	1310.8	1274.4	1232.1	1183.0	1126.0	1058.9	979.5
	5587.7	1401.5	1382.6	1359.4	1331.7	1299.2	1261.2	1217.0	1165.7	1105.7	1035.2
	5655.8	1411.0	1394.8	1374.7	1350.2	1321.2	1287.0	1247.4	1201.2	1147.3	1084.3
	5723.9	1418.5	1404.9	1387.7	1366.3	1340.4	1310.1	1274.3	1232.7	1184.3	1127.9
Loads	***5792.1***	1423.9	1413.2	1398.5	1380.2	1357.6	1330.4	1298.5	1261.2	1217.6	1167.0
(MW)	***5860.2***	1427.4	1419.3	1407.5	1391.8	1372.2	1348.2	1319.8	1286.2	1247.2	1201.6
	5928.4	1428.9	1423.5	1414.4	1401.3	1384.6	1363.8	1338.4	1308.7	1273.5	1232.5
	5996.5	1428.4	1425.7	1419.2	1409.0	1394.9	1377.1	1354.9	1328.4	1297.0	1260.3
	6064.7	1425.9	1425.9	1422.1	1414.6	1403.3	1388.2	1369.1	1345.6	1317.8	1284.7
High	***6150.5***	1421.4	1424.2	1423.0	1418.2	1409.6	1397.2	1381.0	1360.6	1335.9	1306.7

5.2. EQM and VQM

EQM and VQM for bus 8 can be computedwithone of Table II andIII and IV,that contain information of Q-margins and the corresponding probabilities in two identical size matrices. Figure 6 shows the EQMsat bus 8 in three different dependence conditionsbetween the load and wind generation: independent, positively dependent and negatively dependent, respectively.

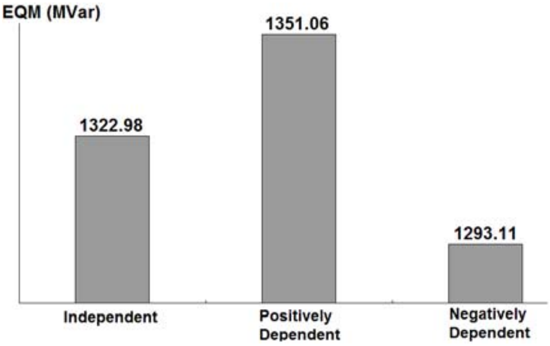


Figure 6. EQMs at Bus 8

Figure 7 shows the distributions of Q-margins in three dependence conditions to calculate the variance of the Q-margins (VQM). Note that Q-margins in positive dependence are more concentrated than the others. This is because a significantly narrow range of Q-margins, from 1306Mvar (case 99) to 1390Mvar (case 00) are picked up from the Q-margin matrix in the computation process of EQM.

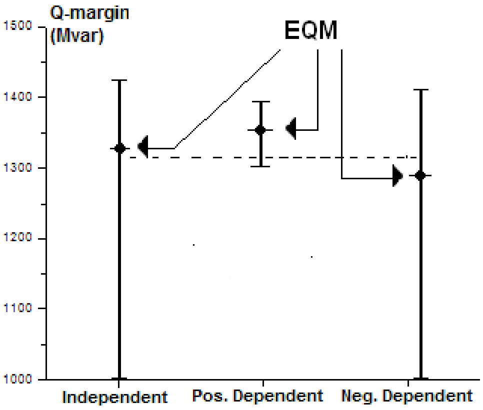


Figure 7. Three Distributions of Q-margins at Bus 8

Different EQMs and VQMs computed from different dependence matrices represent the quantitative impact of uncertain wind generation on the system voltage stability. The uncertainty dependence matrices are arbitrarily given. How dependent the load and the wind generation are will affect the operational decisions in the end. Therefore, EQM and VQM can be utilized by the system operatorswho attempt to make sophisticated operational decisions considering uncertainties of the load and wind generation.

6. CONCLUSION

This paper presents a probabilistic approach for assessment of reactive power reserve of a power system with large-scale uncertain wind generation. A matrix structure is proposed so that the uncertainty relationship between the load and wind generation can be modeled and implemented. With the proposed method, quantitative information about the uncertainties, such as EQM and VQM can be obtained. Such quantitative information can be utilized to make sophisticatedoperational decisions under uncertainty. Unlike Monte Carlo simulation methods, the proposed approach is able to provide more insights on the inter-relations of the load, wind generation, and reactive power reserve. The proposed method can be a tool considering uncertainty and dependencyin other on-going system estimation applications. Numerical simulation with the IEEE 39-bus system demonstrates how the proposed method is implemented and what the computed EQM and VQM represent.

ACKNOWLEDGEMENTS

This work was supported by Arkansas Center for Energy, Natural Resources, and Environmental Studies (ACENRES) research grants, Arkansas Tech University.

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