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An Application of Ulam-Hyers Stability in DC Motors

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ABSTRACT

In this paper, a generalization to nonlinear systems is proposed and applied to the motordynamic, rotor model and stator model in DC motor equation. We argue that Ulam-Hyers stability concept is quite significant in design problems and in design analysis for the class of DC motor's parameters. We prove the stability of nonlinear partial differential equation by using Banach's contraction principle. As an application, the Ulam-Hyers stability of DC motor dynamics equations is investigated. To the best of our knowledge this is the first time Ulam-Hyers stability is considered from the applications point of view.

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1. INTRODUCTION

In 1940, S.M. Ulam [15] proposed the following question concerning stability of group homomorphisms:Under what condition does there exist an additive mapping near an approximately additive mapping? The problem for the case of approximately additive mappings was solved by D. H. Hyers [7] whenthe groups were replaced byBanach spaces. The result of Hyers' study was generalized by Th. M. Rassias [13]. The stability phenomenon that was introduced and proved by Rassias in his paper was called the Hyers–Ulam(HU) stability. G.L. Forti [5] and P. Gavruta [6] have generalized the result of Rassias, which permitted the Cauchy difference to become arbitrarily unbounded. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [3], [8], [12] and [16]).

UH stability guarantees that there is a close exact solution. This is quite useful in many applications e.g. numerical analysis, optimization, electrical motors etc., where finding the exact solution is quite difficult. It also helps, if the stochastic effects are small, to use a deterministic model to approximate a stochastic one. The DC motor control system is a typical example of control systems in which the undesirable impacts of time delays on the system dynamic are observed [4]. DC motor control systems are stable systems in general when time delays are not considered. However, inevitable time delays may destabilize the closed-loop system when the DC motor is controlled through a network [2]. For this reason, time delays must be considered in the process of a controller design, and methods need to be developed to compute the delay margin defined as the maximum amount of time delay for a stable operation. The description of the system stability boundary in terms of time delay also helps us design an appropriate controller for cases in which uncertainty in network-induced delays is unavoidable. To the best of our knowledge, the stability of networked control DC motor speed control systems has not been comprehensively analyzed, and in particular, the description of the stability boundary in terms of the delay margin for a broad range of controller gains has not been reported in the literature. The dynamic models developed in articles do not take into account most of the parameters that may be present in the electricalmachines. Traditionally, most of the models have ignored these phenomenon causing, in certain cases, inaccuracies in the control strategies based on these models. Consequently, in order to develop a model as close as possible to the real machine, all parameters should be considered?Since, the existence and variation of these parameters can be effective in the performance of electrical machines. We prove this claim by using the stabilityof the first order nonlinear partial differential equations. The armature winding resistant and inductance will have varietyin the DC machines model equation. In general, this analysis is oriented to low power machines, since they present more significant power loss than higher power machines. These observations considerably reduce the complexity and cost of electrical machines, controllers and peripherals.

In section 2, we try to use the DC motor dynamics equations and solve them with the two methods of simulation and mathematics to explore the effects of machine parameters. Indeed, the permanent magnet DC motor model is chosen according to its good electrical and mechanical performance compared to other DC motor models. The DC motorsare driven by applied voltage. In section 3, we prove the stability of the following first order nonlinear partial differential equation

$$\theta_x(x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n, \theta(x_1, x_2, \dots, x_n)).$$

Then we employ the result to establish DC motor dynamics equations. In fact, we show that all variables can be effective in such motors.

2. DYNAMIC MODEL OF THE DC MOTOR

In this section we try to use the DC motor dynamics equations and solve them with the two methods of simulation and mathematics to explore the effects of machine parameters.DC machines are characterized by their versatility. By means of various combinations of shunt-, series-, separately-, and permanent magnet-excited field windings they can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady-state operations.

The characteristic equations of motion for DC motors are as follows:

$$V = L\frac{d\,I}{d\,t} + R\ I + k_b\dot{\theta} \tag{1}$$

$$M\ddot{\theta} = k_T I - v\dot{\theta} - \tau \tag{2}$$

where *V* is the voltage applied to the motor *L*, is the motor inductance, *I* the current through the motor windings, *R* the motor winding resistance, k_b the motor's back electromagnetic force constant,˙ the rotor's angular velocity, the rotor's moment of inertia, k_T the motor's torque constant, the motor's viscous friction constant, and the torque applied to the rotor by an external load [2].

2.1. Simulation example.

From the equations (2.1) and (2.2), we can construct the model with environment MATLAB 7.8 (R2009a) in Simulink. The model of DC motor in Simulink is shown in Fig 1. The various parameters of the DC motor are shown in Table 1.

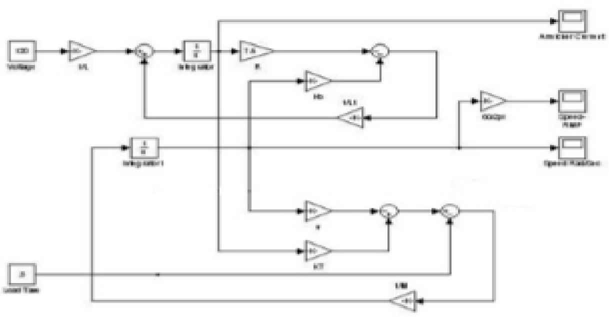

Figure 1. Model of the DC Motor in Simulink

Table 1. Parameters of the DC Motor

$V = 1\ 2\ [V]$	$M = 0.0\ 0\ 0\ 7[K\ gm\hat{}2]$
$R = 7.6\ [\Omega]$	$k_b = 0.6\ 4\ [N.m]$
$L = 6\ 92\ [mH]$	$k_T = k_b$
$v = 0.0\ 0\ 0\ [N7m.S\ s/R\ a]d$	

We apply a voltage source to motor's terminal and mechanical load (a torque) to its rotor. Figure 2 shows the velocity and armature winding current for this motor running at 120 Volts.

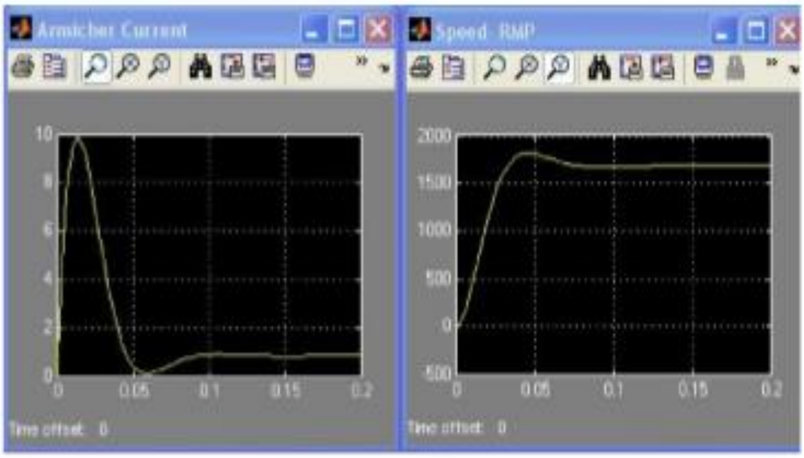

Figure 2.Simulation of transient behavior of the DC motor.

The figure shows the resulting current and angular velocity suddenly and then constant for other second's period. After changing ±5 %in armature winding resistance and run again Simulink figure 3 showing not increased or decreased in current and angular velocity.

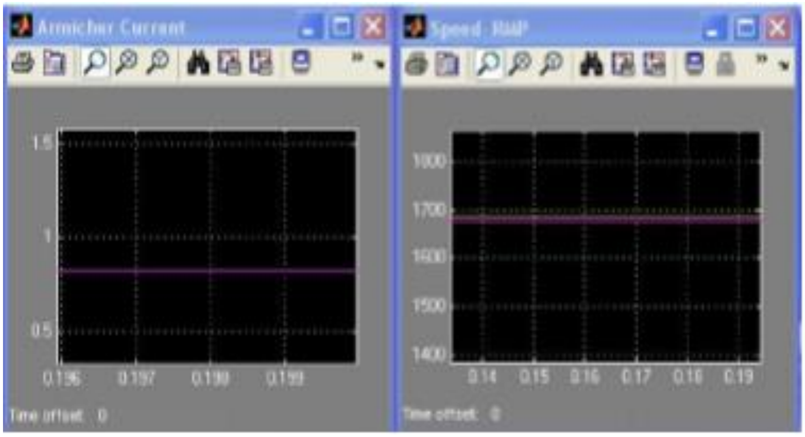

Figure 3.Simulation of behavior of the DC motor after changing

±5%inarmature winding resistance even with the constant zooms in

These changes wereimplemented on theinductanceand much less effectoncurrent and angular velocitywas observed.These resultsshown in Figure 4.

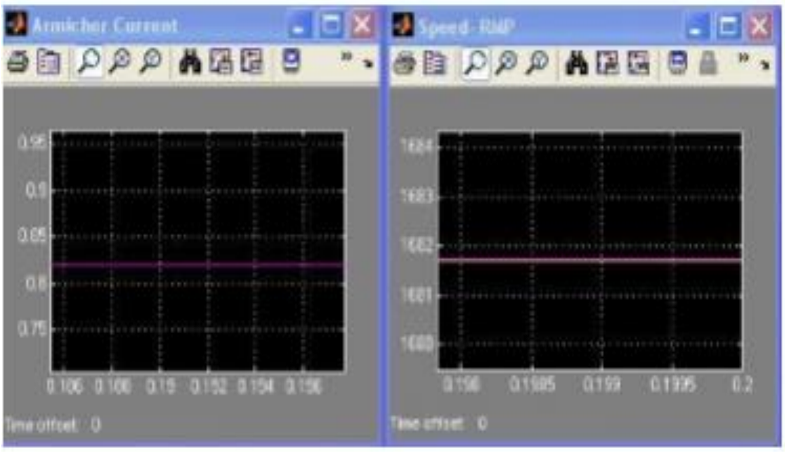

Figure 4.Simulation of behavior of the DC motor after changing

±5%in armature winding inductance even with the constant zoo min

3. STABILITY OF DIFFERENTIAL EQUATIONS

Ulam-Hyers stability studies the following question: Supposeone has a function *y(t)* which is close to solve an equation. Is there an exact solution *x(t)* of the equation which is close to *y(t)?* Mathematically, the following system can be studied

$$\frac{dx}{dt} = f(x). \tag{3.1.}$$

The system (3.1) is Ulam-Hyers (UH) stable if it has an exarct solution and if for each $\epsilon > 0$ there is $\delta > 0$ such that if $x_a(t)$ a is an apporoximation for the solution of (3.1) then there is an exact *x(t)* of (3.1) which is close to$x_a(t)$, that is

$$\left\| \frac{dx_a}{d\ t} - f(x_a(t)) \right\| < \delta \Longrightarrow \|x(t) - x_a(t)\| < \epsilon \qquad (\forall t > 0).$$

The Hyers-Ulam stability of differential equation $y' = y$ was studied for the first time by Alsina and Ger [1]. After that, this result has been generalized by Takahasi et al. [14] for the Banach

Space-valued differential equation $y' = \lambda\ y$ Jung [9] proved thegeneralized Hyers-Ulam stability of a linear differential equation of the first order (see also [10] and [11]).

In this section we prove the stability of the first order nonlinear partial differential equation

$$\theta_x(x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n, \theta(x_1, x_2, \dots, x_n)).$$

Definition 3.1. Let (X, *d*) be a compete metric space. The mapping T $: X \longrightarrow X$ is called contraction if there exists $\in [0, 1)$ such that

$$d(T(x), T(y)) \leq c\ d(x, y) \quad (x, y \in X).$$

Here and subsequently, $I_n = [a_n, b_n]$ is a closed interval of real numbers ℝ . Throughout this section, we assume that $C^{(1)}(I_1 \times I_2 \times \dots \times I_n)$ is all continuously differentiable functions

$f : I_1 \times I_2 \times \ldots \times I_n \longrightarrow \mathbb{R}$. .For a several variable function $h : I_1 \times I_2 \times \ldots \times I_n \longrightarrow \mathbb{R}$, , we denote $(x_1, x_2, \ldots, x_{j-1}, x_j, x_{j+1}, \ldots, x_n)$ and $(x_1, x_2, \ldots, x_{j-1}, u_j, x_{j+1}, \ldots, x_n)$ by $(x_k)_1^n$ **and** $(x_k^j)_1^n$, , respectively.Consider the following partial first order

$$\theta_x(x_k)_1^n = f((x_k)_1^n, \theta(x_k)_1^n) \tag{3.2}$$

It is easy to check that

$$\theta_0(x_k)_1^n = \theta_0\big(x_1, x_2, \ldots, x_{j-1}, x_0, x_{j+1}, \ldots x_n\big) - \int_{x_0}^{x_j} f\big(\big(x_k^j\big)_1^n, \theta_j(x_k)_1^n\big) du_j$$

is a solution of (3.2) for some $x_0 \in I_j$.

Theorem 3.3.Let : $f : I_1 \times I_2 \times \ldots \times I_n \times \mathbb{R} \longrightarrow \mathbb{R}$ and $\Phi : I_1 \times I_2 \times \ldots \times I_n \longrightarrow (0, \infty)$ be continuous functions and let $g : I_1 \times I_2 \times \ldots \times I_n \longrightarrow [1, \infty)$ be an integrable function. suppose that there exists $0 < \alpha < 1$ such that

$$\int_{x_0}^{x_j} g\big(x_k^j\big)_1^n \Phi\big(x_k^j\big)_1^n du_j < \alpha \Phi(x_k)_1^n \tag{3.3}$$

for some $x_0 \in I_j$ *and*

$$|f((x_k)_1^n, \phi(x_k)_1^n) - f((x_k)_1^n, \psi(x_k)_1^n)| < g(x_k)_1^n |\phi(x_k)_1^n - \psi(x_k)_1^n|$$

or all $x_k \in I_k$. then there exists a unique contunuosly differentiable function $\theta_0 : I_1 \times I_2 \times \ldots \times I_n \longrightarrow \mathbb{R}$, *that* θ_0 is a solution pf (3.2) satisfying.

$$|\theta(x_k)_1^n - \theta_0(x_k)_1^n| < \frac{\alpha}{1-\alpha} \Phi(x_k)_1^n \tag{3.5}$$

or all $x_k \in I_k$, *then there exists a unique continuously differentiable function* $\theta_0 : I_1 \times I_2 \times \ldots \times I_n \longrightarrow \mathbb{R}$ *such that* θ_0 is a solution of (3.2) satisfying

$$|\theta(x_k)_1^n - \theta_0(x_k)_1^n| < \frac{\alpha}{1-\alpha} \Phi(x_k)_1^n \tag{3.6}$$

for all $x_k \in I_k$.

Proof. We wish to make all conditions of Theorem 3.2 For this, define the mapping

$$d : C^{(1)}(I_1 \times I_2 \times \ldots \times I_n) \times C^{(1)}(I_1 \times I_2 \times \ldots \times I_n) \longrightarrow [0, \infty)$$

$$d(\phi, \psi) = \sup_{x_k \in I_k} \frac{|\phi(x_k)_1^n - \psi(x_k)_1^n|}{\Phi(x_k)_1^n}$$

for all $\phi, \psi \in C^{(1)}(I_1 \times I_2 \times \ldots \times I_n)$. It is easy to check that $(C^{(1)}(I_1 \times I_2 \times \ldots \times I_n), d)$ is a complete metric space. Also, define the operator $T : C^{(1)}(I_1 \times I_2 \times \ldots \times I_n) \longrightarrow C^{(1)}(I_1 \times I_2 \times \ldots \times I_n)$ through

$$\mathrm{T}(\phi)(x_1, x_2, \ldots, x_n) = \theta(x_1, x_2, \ldots, x_{j-1}, x_0, x_{j+1}, \ldots x_n) + \int_{x_0}^{x_j} f\big(\big(x_k^j\big)_1^n, \phi\big(x_k^j\big)_1^n\big) du_j.$$

For each $\phi, \psi \in C^{(1)}(I_1 \times I_2 \times \ldots \times I_n)$, we have

$$d(T(\Phi), T(\psi)) = Sup_{x_k \in I_k} \frac{\left|\int_{x_0}^{x_j}\left[f\left(\left(x_k^j\right)_1^n, \phi\left(x_k^j\right)_1^n\right) - f\left(\left(x_k^j\right)_1^n, \psi\left(x_k^j\right)_1^n\right)\right] du_j\right|}{\Phi(x_k)_1^n}$$

$$\leq Sup_{x_k \in I_k} \frac{\int_{x_0}^{x_j} g(x_k)_1^n |\phi(x_k)_1^n - \psi(x_k)_1^n| \; du_j}{\Phi(x_k)_1^n}$$

$$= Sup_{x_k \in I_k} \frac{\int_{x_0}^{x_j} g\left(x_k^j\right)_1^n \Phi\left(x_k^j\right)_1^n \frac{\left|\phi\left(x_k^j\right)_1^n - \psi\left(x_k^j\right)_1^n\right|}{\Phi\left(x_k^j\right)_1^n} du_j}{\Phi(x_k)_1^n}$$

$$\leq Sup_{x_k \in I_k} \frac{\int_{x_0}^{x_j} g\left(x_k^j\right)_1^n \Phi\left(x_k^j\right)_1^n Sup_{x_k \in I_k} \frac{\left|\phi\left(x_k^j\right)_1^n - \psi\left(x_k^j\right)_1^n\right|}{\Phi\left(x_k^j\right)_1^n} du_j}{\Phi(x_k)_1^n}$$

$$= d(\phi, \psi) Sup_{x_k \in I_k} \frac{\int_{x_0}^{x_j} g\left(x_k^j\right)_1^n \Phi\left(x_k^j\right)_1^n du_j}{\Phi(x_k)_1^n}$$

$\leq \alpha \; d(\phi, \psi)$

The above relations show that T is a contractive operator. By Theorem 3.2, T has a unique fixed ϕ_0. Indeed,

$$\phi_0(x_1, x_2, \ldots, x_n) = \theta(x_1, x_2, \ldots, x_{j-1}, x_0, x_{j+1}, \ldots, x_n) + \int_{x_0}^{x_j} f\left(\left(x_k^j\right)_1^n, \phi_0\left(x_k^j\right)_1^n\right) du_j$$

And

$$d(\phi, \phi_0) \leq \frac{1}{1-\alpha} d(T(\phi), \phi) \tag{3.7}$$

For all $\phi \in C^{(1)}(I_1 \times I_2 \times \ldots \times I_n)$ integrating both sidesof (3.5) from x_0 to x_j,

$$\left|\theta(x_k)_1^n - \theta(x_1, x_2, \ldots, x_{j-1}, x_0, x_{j+1}, \ldots, x_n) - \int_{x_0}^{x_j} f\left(f\left(\left(x_k^j\right)_1^n, \theta\left(x_k^j\right)_1^n\right)\right) du_j\right|$$

$$\leq \int_{x_0}^{x_j} \Phi\left(x_k^j\right)_1^n du_j$$

$$\leq \int_{x_0}^{x_j} g\left(\left(x_k^j\right)_1^n\right) \Phi\left(\left(x_k^j\right)_1^n\right) du_j \qquad (g(x_1, x_2, , \ldots, x_n) \geq 1)$$

$$\leq \alpha \Phi(x_k)_1^n.$$

It follows from the above relations that

$$\frac{|\theta(x_k)_1^n - T(\theta)(x_k)_1^n|}{\Phi(x_k)_1^n} \leq \alpha$$

for all $x_k \in I_k$. This impies that

$$d(\theta, T(\theta)) < \alpha \tag{3.8}$$

By (3.7) and (3.8), we obtain

$$d(\theta, \theta_0) < \frac{\alpha}{1-\alpha}$$

Now, by definition of the metric d, we deduce that

$$|\theta(x_k)_1^n - \theta_0(x_k)_1^n| < \frac{\alpha}{1-\alpha}\Phi(x_k)_1^n$$

for all $x_k \in I_k$.

We are going back to the characteristic equations (2.1) and (2.2) 0f motion for DC motors. We consider in Theorem 2.3

$$\Phi(R, L, V, v, \tau, k_b, k_T, t) = e^{-\lambda\ t} \text{and} g(R, L, V, v, \tau, k_b, k_T, t) = \frac{\tau}{2k_T}t$$

where $\lambda = V - \frac{R}{k_T}\tau$ and $t \in \left[\frac{2k_T}{\tau}, \infty\right[$. In this case, $g(R, L, V, v, \tau, k_b, k_T, t) \geq 1$ and we have

$$\int_{R_0}^{R_1} \Phi(R, L, V, v, \tau, k_b, k_T, t)\, g(R, L, V, v, \tau, k_b, k_T, t) d\ R = \frac{\tau}{2k_T}\int_{R_0}^{R_1} t e^{-\lambda\ t} d\ R$$

$$= \frac{\tau}{2k_T}\frac{k_T}{\tau}e^{-V\ t}\left(e^{\frac{R_1}{k_T}\tau\ t} - e^{\frac{R_0}{k_T}\tau\ t}\right) \leq \frac{1}{2}e^{-V\ t}e^{\frac{R_1}{k_T}\tau\ t} = \frac{1}{2}\Phi(R_1, L, V, v, \tau, k_b, k_T, t).$$

On the other hand,

$$\left|\frac{k_T}{M}I - \frac{v}{M}\dot{\theta}_2 - \frac{\tau}{M} - \left(\frac{k_T}{M}I - \frac{v}{M}\dot{\theta}_1 - \frac{\tau}{M}\right)\right| = \left|\frac{v}{M}(\dot{\theta}_2 - \dot{\theta}_1)\right| \leq \frac{\tau}{2k_T}t\left|\dot{\theta}_2 - \dot{\theta}_1\right|.$$

Note that since $\frac{v}{M} \leq 1$, we have $\frac{v}{M} \leq \frac{\tau}{2k_T}t$. Also, we can suppose that $\left|M\ddot{\theta} - (k_T I - v\dot{\theta} - \tau)\right| < \Phi(R, L, V, v, \tau, k_b, k_T, t)$. So, by Theorem 3.3, there exists a unique continuously differentiable function θ_0' which is a solution sharacteristic equations of motion for a DC motor and

$$\left|(\dot{\theta} - \dot{\theta}_0)\right| < \Phi(R, L, V, v, \tau, k_b, k_T, t)$$

In other words,

$$\left|(\dot{\theta} - \dot{\theta}_0)\right| < e^{-\lambda\ t}$$

Letting to reach infinity in the last inequality, we see that the approximate solution can approach to the exact solution. The above relations show that the existence and variation of these parameters can be effective in the performance of electrical machines. Indeed, we proved this fact for variable. For other variables, we can choose the suitable and Φ, and obtain an approximation for ˙.

4. CONCLUSION

One can conclude that Ulam-Hyers stability concept is quite significant in realistic problems in parameter analysis and design of DC motors. A generalization to nonlinear systems is proposed and applied to the type of motor equation. The stability of nonlinear partial differential equation by using Banach's contraction principle is proved and applied to finding the best DC motor parameters such as resistance and winding parameters. It is important to notice that there are many applications for UH stability in other topics in the field of electrical motors.

5. CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this article

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