Large-scale wind power grid modelling and stability evaluation using stochastic approaches

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Article Info

Article history:

Received Mar 12, 2022
Revised Aug 14, 2022
Accepted Aug 18, 2022

Keywords:

Lyapunov function
Renewable energy
Stability analysis
Stochastic modelling
Transition matrix
Transition rate

ABSTRACT

Due to dwindling fossil fuel reserves, there is a global demand to increase the level of low-carbon based renewable energy resources (RERs) for electric power generation. Coupled with concerns that emissions from fossil fuels is leading to climate change with possible disastrous consequences, effort is seriously under way to discover the probable usage of RERs on large-scale without being integrated into an existing power grid. It is envisaged that such a large-scale RER power grid will operate solely on its own and expected to be stable and reliable comparable to a conventional power grid. The impact of wind power generation as part of the electric power grid is no longer negligible. Wind energy generation is one of the most established renewable energy resources to help ensure low carbon-based renewable energy (RE) self-sufficiency, yet it is also one of the most volatile RERs. Despite its disadvantages, wind power generation is expected to continue its strong growth in the coming years as result of high interest in clean energy to curb the global warming. Various studies are looking at the prospects for solely RER power grids for usage on large-scale. However, the issue has been the stability and the reliability of such power grids. Variability of power output, intermittency and load mismatch are wind farms' unique characteristics potentially harmful to grid voltage stability. In response to this problem, A large-scale wind power system was modelled using stochastic approach and the results analyzed using Lyapunov method, matrix exponential and Hurwitz criterion to ascertain the stability behavior of an entirely 100% RE grid being envisaged for the near future.

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1. INTRODUCTION

To achieve the year 2030 agenda for sustainable development (SD) as well as the Paris agreement on climate change, energy availability and affordability is key. Effort is being made by several countries around the world towards the search for sustainable and renewable energy (RE) alternatives to supplement their energy requirement due to factors such as the increasing demand for energy, the decline in fossil fuel reserves, CO_2 reduction and global climate change [1]. The effects of the rise in global temperature is of great concern. Greenhouse gas (GHG) emissions which is as result of human activities is seriously affecting the Earth's climate. It has been reported that the Earth's climate has warmed up between the range of 0.8 °C to 1.2 °C since 1882 and may reach 1.5 °C between the year 2030 and the year 2052 unless drastic measures are put in place. One of the most visible effects of global warming has been the rapid decrease in glaciation experienced over the years. It is being speculated in certain quarters that the rising global temperature being experienced in modern times is due, in part, to atmospheric pollution arising from human activities, which is fuelling the Earth's GHG effect. Although the GHG effect is essential to the well-being of mankind, excessive GHG levels above their natural norm, the corresponding additional warming could threaten the sustainability of the planet as a whole. The prediction is that is as global warming progresses, sea levels will rise by over 400 mm by the year 2080 due to the combined effects of thermal expansion of the oceans and melting of the polar ice. This will negatively impact livelihood of mankind, with an over 80 million people threatened with flooding in the low-lying territories [2], [3].

Electricity generation from RERs was said to have increased by over 8% in the year 2021, accounting for more than half of the increase in overall electricity production worldwide. RERs were expected to provide 30% of electricity generation worldwide by the end of the year 2021, their biggest share in the power generation mixes since the beginning of the Industrial Revolution. The largest contribution to that growth was expected to come from solar photovoltaic (PV) and wind. Electricity generation from wind is expected to grow by 275 terawatt-hours, or around 17%, as compared to that of the year 2020. Electricity generation from solar PV is expected to increase by 145 terawatt-hours, an increment of about 18% as compared to that of the year 2020. Their combined output was expected to have hit about 2800 terawatt-hours at the end of the year 2021 [3], [4]. A number of studies are looking at the prospects for solely renewable energy (RE) power grids, on large-scale. RERs are likely to account for about 95% of the net increase in global electric power supply by the year 2025 [5], [6]. The simplest way to meet these ambitions is to reduce the emissions from fossil fuels by deploying large-scale RERs [6].

The literature provided interesting expose regarding stochastic modelling of large-scale RERs integrated electric power grids. However, stochastic modelling of entirely RER electric power grid on large-scale was not found. The recent ones on RERs integrated large-scale electric power grids include: [7] established stochastic transient stability analysis method by introducing stochastic differential equations into the traditional transient stability analysis method. Utilizing the theory of probability and statistics, the Monte Carlo simulation was utilized to improve the numerical simulation method for transient stability evaluation. Using formulated impedance network model of large-scale renewable energy bases with frequency coupling, the s-domain nodal admittance matrix (SNAM) was formulated [8]. The stability analysis of a 10-node system containing 4 PMSG-based wind farms was carried out using the SNAM method. Kessywa *et al.* [9] conducted dynamic modelling and control study for the assessment of large-scale wind and solar photovoltaic integration into existing power system. The model was implemented in MATLAB-based transient stability analysis toolbox in order to analyses the dynamic response of the renewable energy sources. The developed model assumed that the converter is the only means through which the renewable energy resources interfaced the existing the power system. A novel method was proposed for a wind power output scene simulation [10]. A genetic algorithm (GA) K-means was used to divide the wind farm into clusters where K is the cluster number. The wind power output of each cluster was computed using the wind turbine model. Power output scenes were simulated based on Markov chain Monte Carlo (MCMC) method. A probabilistic analysis approach was used to investigate the impact of stochastic uncertainty of grid-connected wind generation on power system small-signal stability. The approach can compute the probabilistic density function (PDF) of critical eigenvalues of a large-scale multiple source wind integrated power system [11].

Model-based and measurement-based analyses are the two main approaches for determining an electric power system's behavior under operation. Measurement-based analysis is a real-time monitoring approach for electric power systems. It differs from model-based analysis. It employs phasor measurement units (PMUs) which are high-speed sensors that measure voltage and current synchro-phasors of the electric power system with high accuracy to obtain the necessary information from the electric power system. Measurement-based analysis provides accurate data and the prediction of the stability behavior of an electric power system based on synchro-phasor technology. When model-based analysis approach is used to determine the stability of an electric power system, the electric power system is described using mathematical models. In situations where the effect of probabilistic uncertainty is considered, stochastic dynamical models play an important role in such analyses. Among the analysis approaches that can be applied to the mathematical model include Lyapunov method, eigenvalue analysis, and matrix exponential method and Routh-Hurwitz stability criterion [12], [13]. For this research, matrix Lyapunov function, matrix exponential and Hurwitz criterion were used to examine the stability behavior of a large-scale wind power system modelled using stochastic approach

Wind energy is expanding steadily and significantly over the world as a result of the demand for more low-carbon RERs in the generation of electric power. One of the important technologies that has been around for a while, can be placed in utility-scale facilities with large installed capacity, and can compete competitively with traditional generation sources is wind power [14]. A by-product of solar energy is wind. Air moves as a result of atmospheric pressure gradients, which cause wind. From high-pressure areas to low-pressure areas, wind blows. The higher the wind speed and the greater the amount of wind power that can be harvested from the wind by a wind turbine (WT), the greater the air pressure gradient. It is on record that the total solar power received by the earth is around 1.8×10^{11} MW. Only 2% (3.6×10^{9} MW) of the solar energy is converted into wind energy, and approximately 35% of wind energy is dissipated within 1000 m of the earth's surface. The available wind power that can be converted into other forms of energy is estimated to be around 1.26×10^{9} MW which is about 20 times the rate of the present global energy consumption. Wind energy in principle could meet the entire energy needs of the world if harnessed effectively and efficiently [15]. The wind speed that is available has a significant impact on the power output of a WT. The height of the terrain, time of year, season, and location all affect the wind speed. However, it is a resource that exhibits significant variability at almost all-time scales, from the length of turbulent gusts to daily and seasonal cycles to long-term changes brought on by climatic conditions. At these time scales, various challenges, such as issues with stability, power quality, and reliability, arise [14]. The wind speed affects the WT's efficiency and safety. Three sets of wind speeds have been established by the International Electrotechnical Commission (IEC) for use in assessing the effectiveness and dependability of a WT. As indicated in Figure 1, the WT manufacturers typically offer these three speeds for any WT they produce [15]–[18]. A WT's performance is indicated by its power curve. The power output is zero in the initial zone where the wind speed is below a cut-in speed, or threshold minimum. There is a quick increase in power output in the second region, which is located between the cut-in and the rated speed. Until the cut-off speed is reached, the third zone produces a constant output (rated). Beyond this speed (region 4), the turbine is shut off to safeguard its internal parts from strong winds; as a result, no electricity is produced in this area. A power system comprising only RERs only should be possible to operate in the near future on large-scale due to the dwindling fossil fuel resources. However, the challenge to overcome in this regard is the stability and reliability of such future large-scale power systems. This research aims at developing a model to assess the stability of such future RERs power systems.

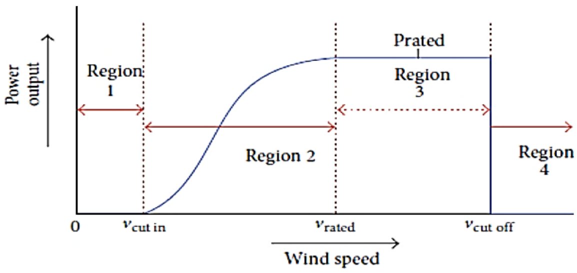

Figure 1. Typical wind power curve of a wind turbine

2. MODEL FORMULATION

This section describes how the model was formulated.

2.1. Wind power generation system modelling based on chapman-kolmogorov (C-K) equation

In stochastic model, the knowledge of the state variables is subject to uncertainties due to some inherent randomness. This type of model is suitable for applications where the state information cannot be perfectly predictable. Markov processes are often used to model randomness and have no memory. The differential form of the Chapman-Kolmogorov equation is the Chemical Master Equation (CME) for jump processes or diffusive processes. The CME describes the evolution of the probability distribution of states of a continuous-time Markov process and is commonly used to describe stochastic physical, chemical, or biological systems. Jump processes are characterised by discontinuous motion; thus, there is a bounded and non-vanishing transition probability per unit time. The ME is a stochastic model and is derived from the fundamental assumption on the dynamic properties of the underlying stochastic process referred to as the Markov property [19]. As a result of wind speed intermittency or variations, the power generation into the power grid is stochastic or random in character impacting the stability and reliability of the power grid. The fluctuation of the renewable power generation can be regarded as random variables which are real-valued functions defined on a sample or state space with the assignment of possible probabilities to the possible values of the random variables [20]. Due to the large-scale nature of the renewable power grid being considered, continuous random variable is used which can be easily handled analytically as compared to discrete random variables. Again, for large-scale modelling, continuous random variables are preferred [21].

The model formulation is based on the modified wind power curve given in Figure 2. Mid speed was introduced between the cut-in speed and the rated output speed for the purposes of the modelling for the realization of four transition states. For this study, the intermittency nature and the changing pattern of the wind speed is transformed into a transition diagram as given in Figure 3 having four states. Transitions between the different states occur from time to time. One can represent the transitions among the various states as random events that do occur at some rates. Each transition state is assigned a probability, p and in between the states is the transition rates represented by ω and δ. Based on Figure 3, the 12 possible transition states are as given by Table 1. A stochastic process is a process that evolves probabilistically through various states attained at various times from a well-defined initial state x_0 at t_0. The probability density function (pdf) of the process depends on their respective states and times at particular moment.

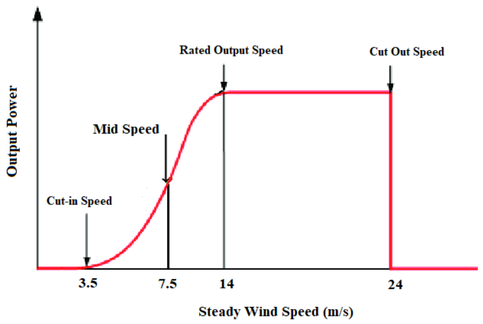

Figure 2. Wind power curve of a wind turbine with mid-speed introduced

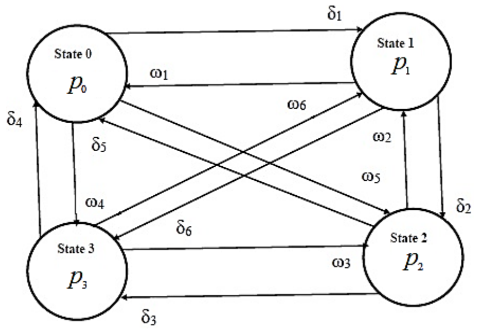

Figure 3. State transition diagram for wind turbine systems

Table 1. Wind speed variation transition states

S/No.	Current state	Next state	Transition rate	S/No.	Current state	Next state	Transition rate
1	0	1	δ_1	7	2	0	δ_5
2	0	2	ω_5	8	2	1	ω_2
3	0	3	ω_4	9	2	2	δ_3
4	1	0	ω_1	10	3	0	δ_4
5	1	2	δ_2	11	3	1	ω_6
6	1	3	δ_6	12	3	2	ω_3

The probability that the process will be at state x_1 at time, t_1; then progress to state x_2, at time, t_2 then goes through all other subsequent configurations or states to be at state x_n at time t_n, given that it started from state x_0 at time t_0 is given by (1):

$$P_n|_1 = ((x_n, t_n), (x_{n-1}, t_{n-1}), (x_{n-2}, t_{n-2}) \ldots\ldots\ldots (x_1, t_1)|(x_0, t_0)) \quad (1)$$

The probability of the system being at state, x_1 at time t_1, multiplied by the probability of it being at state, x_2 at time, t_2 given that it has been at state, x_1 at t_1, and at state x_0, at time, t_0 respectively can be expressed as [22], [23]:

$$P_{2|1}((x_2, t_2), (x_1, t_1)|(x_0, t_0)) = P((x_1, t_1)|(x_0, t_0))P_{1|2}(x_2, t_2)|(x_1, t_1), (x_0, t_0)) \quad (2)$$

If all such probabilities are known, the stochastic process is fully specified [22], [23]. The ME for jump processes is given by (3).

$$\frac{\partial p(x,t)}{\partial t} = \int (w(x|x')p(x', t) - w(x'|x)p(x, t))dx' \quad (3)$$

Where: $p(x, t)$ = probability of being in state x at time, t; $w(x|x')$ = transition probability of moving from state x' to state x per unit time.

One-step stochastic process is continuous-time Markov process whose range consists of the integer k, and whose transition probability per unit time, ω_{nm} permits only jumps between adjacent sites [24]. Thus,

$$\omega_{km} = r_m \delta_{k,m-1} + g_m \delta_{k,m+1},^{(m \neq k)}$$

$$\omega_{km} = 1 - (r_k + g_k) \quad (4)$$

where, r_k = probability per unit time that, being in state k, a jump occurs to site $k - 1$, g_k= probability per unit time that, being in state k, a jump occurs to state $k + 1$.

One-step transition probabilities are also given by conditional probabilities [25]:

$$Q_{ij}(n) = Q(X_{n+1} = j/X_n = i); n = 0,1,2\ldots.. \quad (5)$$

One-step transition probabilities combined in matrix of one-step transition probabilities, Q is given by:

$$P = \begin{pmatrix} q_{00} & q_{01} & q_{02} & \cdots \\ q_{10} & q_{11} & q_{12} & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ q_{i0} & q_{i1} & q_{i2} & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{pmatrix} \quad (6)$$

where, Q_{ij}= probability of a transition from state *i* to state *j* in one step or one time unit or in one jump.

A wind turbine (WT) transitioning from one state to another can be regarded as a jump process or one-step transition. For this modelling, every transition state is taken to be positive since the WT is not supposed to be stationary hence, a current state depends on all other previous states which can be treated as a union of disjoint intervals based on the theory and concept of continuous random variable. For a continuous random variable *X* with probability density function (PDF) $fx(x)$, the following holds [20]:

$$f_X(x) \geq 0 \text{ for all } x \in \mathbb{R} \quad (7)$$

$$\int_{-\infty}^{\infty} f_X(u)\, du = 1 \quad (8)$$

$$P(a < X \leq b) = F_X(b) - F_X(a) = \int_a^b f_X(u)du \quad (9)$$

More generally, for a set $A, P(X \in A) = \int_A f_X(u)du$ (10)

To determine the probability of a continuous random variable *X*, for instance, $P(X \epsilon$ [1, 2] U [5, 6]; one can write [20]:

$$P(X \in [1,2] \cup [5,6] = \int_1^2 f_X(u)du + \int_5^6 f_X(u)du \quad (11)$$

Based in (3), the ME for jump processes is modified and applied to wind turbine (WT) power generation system which is given by (12):

$$\frac{\partial p(x,t)}{\partial t} = \int (w(x|x')p(x',t) + w(x'|x)p(x,t))dx' \quad (12)$$

Assuming the system is at state x (state 1 or initial state) at time, t moving to state $x-1$ (state 2 or next state) at time, $t + \Delta t$. The probability of moving from state *x* (state 1 or initial state) to state $x-1$ (state 2 or next state) is given by $\Omega\Delta t$. The transition must occur during the time, Δt. For the transition to occur: i) The WT power generation system must be at state *x* (state 1 or initial state) with the probability, $p_x(t)$; and ii) Transition must occur from state *x* (state 1 or initial state) to state $x-1$ (state 2 or next state) with the probability, $\Omega\Delta t$, where Ω = transition rate.

The probability of transitioning from state *x* (state 1 or initial state) to state $x-1$ (state 2 or next state) between the times t, $t+\Delta t$ is the product of the two probabilities given by $px(t)\Omega\Delta t$. During that small time interval, Δt, only one change in state must occur, either *x* – 1 (decrement in wind speed) or $x+1$ (increment in wind speed). Transitioning to the next state depends on the previous state and the transition rate. Based on the above, dynamical equations can be developed for state *x* (state 1 or initial state), state x– 1 (state 2 or next state) or $x+1$ state (another possible state 2 or next state) as follows based on the rules of conditional probability [20], [22]. This can be translated into probability terms as follows:

$$P_{(x)}(t+\Delta t) = P_{(x)}(t) + P_{(x)}(t)\Omega\Delta t \quad (13)$$

That of state $(x-1)$ transition is given by:

$$P_{(x-1)}(t+\Delta t) = P_{(x-1)}(t) + P_{(x-1)}(t)\Omega\Delta t \quad (14)$$

And that of state $(x+1)$ is also be given by:

$$P_{(x+1)}(t+\Delta t) = P_{(x+1)}(t) + P_{(x+1)}(t)\Omega\Delta t \quad (15)$$

Dividing through in (13), (14) and (15) respectively by Δt and taking the limit as $\Delta t \rightarrow 0$, in (13), (14) and (15) becomes:

$$\frac{dp_x}{dt} = \Omega p_x \tag{16}$$

$$\frac{dp_{(x-1)}}{dt} = \Omega p_{(x-1)} \tag{17}$$

$$\frac{dp_{(x+1)}}{dt} = \Omega p_{(x+1)} \tag{18}$$

Generalizing and applying the principle to the entire four states with twelve transition diagrams of Figure 3 (with the transitions given in Table 1) which represents the movement pattern of WTs due to fluctuations in wind speed with 1, 2, 3, …………,n number of wind farms comprising a number of WTs; the dynamical ME in stochastic transitional probability matrix-vector form:

$$\frac{d}{dt}\begin{pmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{pmatrix} = \begin{pmatrix} -(\delta_{11}+\omega_{51}+\omega_{41}) & \omega_{11} & \delta_{51} & \delta_{41} \\ \delta_{11} & -(\omega_{11}+\delta_{21}+\delta_{61}) & \omega_{21} & \omega_{61} \\ \omega_{51} & \delta_{21} & -(\delta_{51}+\omega_{21}+\delta_{31}) & \omega_{31} \\ \omega_{41} & \delta_{61} & \delta_{31} & -(\delta_{41}+\omega_{61}+\omega_{31}) \end{pmatrix}\begin{pmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{pmatrix} = \begin{pmatrix} -(\delta_{12}+\omega_{52}+\omega_{42}) & \omega_{12} & \delta_{52} & \delta_{42} \\ \delta_{12} & -(\omega_{12}+\delta_{22}+\delta_{62}) & \omega_{22} & \omega_{62} \\ \omega_{52} & \delta_{22} & -(\delta_{52}+\omega_{22}+\delta_{32}) & \omega_{32} \\ \omega_{42} & \delta_{62} & \delta_{32} & -(\delta_{42}+\omega_{62}+\omega_{32}) \end{pmatrix}\begin{pmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{pmatrix}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{d}{dt}\begin{pmatrix} p_{1n} \\ p_{2n} \\ p_{3n} \\ p_{4n} \end{pmatrix} = \begin{pmatrix} -(\delta_{1n}+\omega_{5n}+\omega_{4n}) & \omega_{1n} & \delta_{5n} & \delta_{4n} \\ \delta_{1n} & -(\omega_{1n}+\delta_{2n}+\delta_{6n}) & \omega_{2n} & \omega_{6n} \\ \omega_{5n} & \delta_{2n} & -(\delta_{5n}+\omega_{2n}+\delta_{3n}) & \omega_{3n} \\ \omega_{4n} & \delta_{6n} & \delta_{3n} & -(\delta_{4n}+\omega_{6n}+\omega_{3n}) \end{pmatrix}\begin{pmatrix} p_{1n} \\ p_{2n} \\ p_{3n} \\ p_{4n} \end{pmatrix} \tag{19}$$

In (19) is modified accordingly based in (12):

$$\frac{d}{dt}\begin{pmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{pmatrix} = \begin{pmatrix} (\delta_{11}+\omega_{51}+\omega_{41}) & \omega_{11} & \delta_{51} & \delta_{41} \\ \delta_{11} & (\omega_{11}+\delta_{21}+\delta_{61}) & \omega_{21} & \omega_{61} \\ \omega_{51} & \delta_{21} & (\delta_{51}+\omega_{21}+\delta_{31}) & \omega_{31} \\ \omega_{41} & \delta_{61} & \delta_{31} & (\delta_{41}+\omega_{61}+\omega_{31}) \end{pmatrix}\begin{pmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{pmatrix} = \begin{pmatrix} (\delta_{12}+\omega_{52}+\omega_{42}) & \omega_{12} & \delta_{52} & \delta_{42} \\ \delta_{12} & (\omega_{12}+\delta_{22}+\delta_{62}) & \omega_{22} & \omega_{62} \\ \omega_{52} & \delta_{22} & (\delta_{52}+\omega_{22}+\delta_{32}) & \omega_{32} \\ \omega_{42} & \delta_{62} & \delta_{32} & (\delta_{42}+\omega_{62}+\omega_{32}) \end{pmatrix}\begin{pmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{pmatrix}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{d}{dt}\begin{pmatrix} p_{1n} \\ p_{2n} \\ p_{3n} \\ p_{4n} \end{pmatrix} = \begin{pmatrix} (\delta_{1n}+\omega_{5n}+\omega_{4n}) & \omega_{1n} & \delta_{5n} & \delta_{4n} \\ \delta_{1n} & (\omega_{1n}+\delta_{2n}+\delta_{6n}) & \omega_{2n} & \omega_{6n} \\ \omega_{5n} & \delta_{2n} & (\delta_{5n}+\omega_{2n}+\delta_{3n}) & \omega_{3n} \\ \omega_{4n} & \delta_{6n} & \delta_{3n} & (\delta_{4n}+\omega_{6n}+\omega_{3n}) \end{pmatrix}\begin{pmatrix} p_{1n} \\ p_{2n} \\ p_{3n} \\ p_{4n} \end{pmatrix} \quad (20)$$

Where the first subscript represents the transition rate number, and the second subscript also represents the wind farm number. The state stochastic transition probability matrix for the first wind farm is therefore given by:

$$U = \begin{pmatrix} (\delta_{11}+\omega_{51}+\omega_{41}) & \omega_{11} & \delta_{51} & \delta_{41} \\ \delta_{11} & (\omega_{11}+\delta_{21}+\delta_{61}) & \omega_{21} & \omega_{61} \\ \omega_{51} & \delta_{21} & (\delta_{51}+\omega_{21}+\delta_{31}) & \omega_{31} \\ \omega_{41} & \delta_{61} & \delta_{31} & (\delta_{41}+\omega_{61}+\omega_{31}) \end{pmatrix} \quad (21)$$

The transition rate for the wind farms, which varies between 0.1 to 1.0, the least being 0.1 and the maximum being 1.0. The maximum being 1.0 is assumed for the wind farm model. In (21) results in stochastic transition probability matrix, U:

$$U = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix} \quad (22)$$

Even though large-scale WT power generation system with its associated size of state matrix, [U] should be in order of thousand or more, depending on the number of wind farms being considered that goes with its many eigenvalues, it is not necessary to compute all these eigenvalues [26].

2.2. Model assumptions

The following assumptions were made during the development of the model:

- The wind speed variation is a function of the weather conditions only;
- A transition to the next state may occur at any point in time;
- Only one transition can occur at a time (1-step transition);
- The transition to another state is gradual over time (not immediate);
- A transition from one state to the next state for all WTs constituting a wind farm occurs at the same time
- The transition rates vary between 0.1 – 1.0;
- The wind power plants are spread geographically to deal with the issue of short-term variability;
- The WTs operate in variable-speed mode, and the control system regulates the rotor speed to obtain peak efficiency in fluctuating winds;
- The wind farms are sited in areas with high average wind speeds; and
- The electrical load model is based on Markov property (current state does not depend on previous state).

2.3. Determination of characteristic equation

The stochastic transitional probability matrix, *U* given by (22):

$$U = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix} \quad (22)$$

The matrix, *U* has non-trivial solution given by (23) which says that the values of λ which satisfies:

$$det|U - \lambda I = 0| \quad (23)$$

Where: λ = unknown eigenvalues of the matrix, *U;* I = *n* × *n* identity matrix. In general, for any $n \times n$ matrix *U*,

$$U - \lambda I = \begin{bmatrix} u_{11}-\lambda & u_{12} & .... & u_{1n} \\ u_{21} & u_{22}-\lambda & .... & u_{2n} \\ .... & ..... & .... & .... \\ u_{n1} & u_{n2} & .... & u_{nn}-\lambda \end{bmatrix} \tag{24}$$

The determinants in (24) is expanded along the first row as follows [27]:

$$Det\,(U-\lambda I) = (u_{11}-\lambda)\ det \begin{bmatrix} u_{22}-\lambda & ..... & u_{2n} \\ ..... & ..... & .... \\ u_{n2} & ..... & u_{nn}-\lambda \end{bmatrix} + ... \tag{25}$$

Based in (24) and (25), (22) is solved as follows for the eigenvalues or characteristic roots of the wind farm 1 (WF-1) made up of a number of wind turbines as follows:

$$\left(\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \right) = 0 \tag{26}$$

$$\begin{bmatrix} 3-\lambda & 1 & 1 & 1 \\ 1 & 3-\lambda & 1 & 1 \\ 1 & 1 & 3-\lambda & 1 \\ 1 & 1 & 1 & 3-\lambda \end{bmatrix} = 0 \tag{27}$$

The determinant in (27) is given by (28):

$$\frac{1.0\lambda^6 - 18.0\lambda^5 - 128.0\lambda^4 - 464\lambda^3 + 912.0\lambda^2 - 928.0\lambda + 384.0}{1.0\lambda^2 - 6.0\lambda + 8.0} \tag{28}$$

Therefore, the characteristic equations of wind farms 1, 2……….n is given by:

$$\begin{array}{l} \lambda_1^4 - 12\lambda_1^3 + 48\lambda_1^2 - 80\lambda_1 + 48 = 0 \\ \lambda_2^4 - 12\lambda_2^3 + 48\lambda_2^2 - 80\lambda_2 + 48 = 0 \\ \vdots \\ \vdots \\ \vdots \\ \lambda_n^4 - 12\lambda_n^3 + 48\lambda_n^2 - 80\lambda_n + 48 = 0 \end{array} \tag{29}$$

2.4. Lyapunov stability analysis approach

To assess the stability of a particular system using energy function approach, the Lyapunov function and its derivative with respect to time is determined. Lyapunov function may include physical variables or the whole state variables of the system under consideration. The quadratic Lyapunov function and its derivative with respect to time are defined as follows for any linear system [28], [29]:

$$\dot{x} = xA \tag{30}$$

Its corresponding Lyapunov function is defined in matrix form as:

$$V(x) = x^T P x \tag{31}$$

Where, P is a real symmetric matrix. Its derivative is given by:

$$\begin{cases} \dfrac{dV}{dt} = \dfrac{\partial V}{\partial x}\dfrac{\partial x}{\partial t} = x^T(A^T P + PA)x \\ = x^T(A^T P + PA)x = x^T Q x \end{cases} \tag{32}$$

Where, x and x^T are the state variables' vector and its transpose vector respectively. For any positive definite Hermitian or symmetric matrix Q, a positive definite Hermitian or symmetric matrix P, satisfying the following Lyapunov matrix equation given by (33) based on (32) [28]–[32].

$$A^T P + PA = -Q \tag{33}$$

Where, A, P, $Q = Q^T \in \mathbb{R}$; holds for some positive definite $Q = Q^T > 0$ and $P = P^T > 0$, then matrix A is stable. Where, Q^T and P^T are the transpose of the matrices Q and P respectively.

To evaluate the stability of any dynamical system using the Lyapunov-based approach, the first step is to choose an appropriate Q matrix, which should be positive definite. If both P and Q are positive definite, then the system works in the stable mode. By defining the P, Q will be determined based on the system state matrix. Alternatively, by defining Q, P can be determined. For this particular study, Q is defined in a specific positive definite manner, then system stability is determined based on the analyzing the matrix, P. The quadratic function, $V(x)$ based on matrix given by (22) is defined as [27]–[29], [33]:

$$\begin{cases} V(x) = x^T P x = [x_1 \ x_2 \ x_3 \ x_4]\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{12} & p_{22} & p_{23} & p_{24} \\ p_{13} & p_{23} & p_{33} & p_{23} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \\ V(x) = p_{11}x_1^2 + 2p_{12}x_1x_2 + 2p_{13}x_1x_3 + 2p_{14}x_1x_4 + p_{22}x_2^2 + 2p_{23}x_2x_3 + 2p_{24}x_2x_4 \\ \qquad + p_{33}x_3^2 + 2p_{34}x_3x_4 + p_{44}x_4^2 \end{cases} \tag{34}$$

Where, x is real vector and P is real symmetric matrix. Based in (33), Q is chosen as positive-definite unit matrix given by:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

The real symmetric n x n matrix P, is also given by:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{12} & p_{22} & p_{23} & p_{24} \\ p_{13} & p_{23} & p_{33} & p_{23} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{bmatrix} \tag{36}$$

Where the matrix P's elements are P_{11}, P_{12}. . .P_{44}. Substituting in (35) and (36) into (33) to obtain the matrix, P expressed by (36):

$$P = \begin{bmatrix} -0.2083 & 0.0417 & 0.0417 & 0.0417 \\ 0.0417 & -0.2083 & 0.0417 & 0.0417 \\ 0.0417 & 0.0417 & -0.2083 & 0.0417 \\ 0.0417 & 0.0417 & 0.0417 & -0.2083 \end{bmatrix} \tag{37}$$

Its associated quadratic Lyapunov function, V(x) based in (34) is given by:

$$V(x) = -0.2083x_1^2 + 0.0834x_1x_2 + 0.0834x_1x_3 + 0.0834x_1x_4 - 0.2083x_2^2 + 0.0834x_2x_3 + 0.0834x_2x_4 \\ + 0.2083x_3^2 + 0.0834x_3x_4 + 0.2083x_4^2$$

The eigenvalues of the matrix, *P* are given by:

$$P(\lambda) = [-0.2500, -0.2500, -02500, -0.0833] \quad (38)$$

2.5. Matrix exponential function analysis approach

For any $n \times n$ square matrix, *A* that can be diagonalized as:

$$A = SDS^{-1} \quad (39)$$

where matrix, *S* consist of eigenvectors of *A*, and *D* is a diagonal matrix with the eigenvalues of *A* along the diagonal, *t* is a real or complex variable; then its matrix exponential is given by:

$$e^{At} = Se^{Dt}S^{-1} \quad (40)$$

The (40) is expanded as follows:

$$e^{A\,t} = \frac{\sum_{i-0}^{\infty} A^i}{i!} = 1 + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \cdots \quad (41)$$

Which is an imitation of the power series:

$$e^x = \sum_{i=0}^{x^i} \frac{(x)^i}{i!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (42)$$

where, A, A^2, A^3, …… are all an $n \times n$ matrix. A Laplace transform, *L* is an operator which takes a function *F(t)* as its input and produces *f(s)* as its input. The inverse Laplace Transform L^{-1} takes *f(s)* as input and produces *F(t)* as output. Therefore,

$$L[e^{At}] = L\left[\sum_{i=0}^{\infty} \frac{(At)^i}{i!}\right] = \{[sI_n - A]^{-1}\} \quad (43)$$

and

$$\Phi(t) = L^{-1}\{[sI_n - A]^{-1}\} = e^{At} \quad (44)$$

$$\Phi(t) = L^{-1}((sI - A)^{-1}) = I + tA + \frac{(tA)^2}{2!} + \cdots \quad (45)$$

$$(sI - A)^{-1} = (1/s)(I - A/s)^{-1} = \frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \cdots \quad (46)$$

where, $(sI - A)^{-1}$ is known as the resolvent of *A*, L and L^{-1} is the Laplace transform and inverse Laplace transform respectively, and I_n is $n \times n$ unit matrix.

In (43) and (44) are applicable to time-varying linear systems and depends on the initial as well as the final time and not just the difference between them All linear systems that are asymptotically stable are also exponentially stable, [33], [34]. Applying in (39), (40), (43) and (44) respectively to the transition matrix given by (22):

$$S = \begin{pmatrix} -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad (47)$$

$$S^{-1} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad (48)$$

$$D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix} \quad (49)$$

Substituting in (47)-(49) into (40) given by:

$$e^{At} = \begin{pmatrix} -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} e^{2t} & 0 & 0 & 0 \\ 0 & e^{2t} & 0 & 0 \\ 0 & 0 & e^{2t} & 0 \\ 0 & 0 & 0 & e^{6t} \end{pmatrix} \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad (50)$$

$$e^{At} = \begin{pmatrix} \frac{3e^{2t}+e^{6t}}{4} & \frac{-e^{2t}+e^{6t}}{4} & \frac{-e^{2t}+e^{6t}}{4} & \frac{-e^{2t}+e^{6t}}{4} \\ \frac{-e^{2t}+e^{6t}}{4} & \frac{3e^{2t}+e^{6t}}{4} & \frac{e^{6t}-e^{2t}}{4} & \frac{e^{6t}-e^{2t}}{4} \\ \frac{e^{6t}-e^{2t}}{4} & \frac{e^{6t}-e^{2t}}{4} & \frac{e^{6t}+3e^{2t}}{4} & \frac{e^{6t}-e^{2t}}{4} \\ \frac{-e^{2t}+e^{6t}}{4} & \frac{e^{6t}-e^{2t}}{4} & \frac{e^{6t}-e^{2t}}{4} & \frac{3e^{2t}+e^{6t}}{4} \end{pmatrix} \quad (51)$$

A quick check to determine the accuracy of the obtained exponential matrix equation is to set $t = 0$ in the right hand side of (51) to obtain a unit or identity matrix, I given by (52) [35]:

$$e^{A(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (52)$$

2.6. Hurwitz stability criterion approach

Hurwitz criterion is an approach used to determine the stability of a system by using the coefficients of the characteristic polynomial equation without counting the roots involved. Given the characteristic polynomial [36]–[38]:

$$p(\lambda) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots\ldots\ldots\ldots + a_1\lambda + a_0 \quad (53)$$

Where $a_0, \ldots\ldots\ldots\ldots, a_n \in \mathbb{R}, a_0, a_n > 0$.

The characteristic polynomial, $p(\lambda)$ with real coefficients is stable or a Hurwitz polynomial if all its zeros have negative real parts. Hurwitz matrix, H for (53) is given by (54) [39], [40]:

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & \ldots\ldots\ldots & 0 \\ a_3 & a_2 & a_1 & 1 & \ldots\ldots\ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & \ldots\ldots\ldots & 0 \\ a_7 & a_6 & a_5 & a_4 & \ldots\ldots\ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & \cdots & \cdots & a_n \end{bmatrix} \quad (54)$$

The stability is determined using the sub-determinants of the characteristic polynomial, $p(\lambda)$ as follows [22], [38]:

$$D_1 = a_1 > 0 \quad (55)$$

$$D_2 = \begin{vmatrix} a_1 a_0 \\ a_3 a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3 > 0 \quad (56)$$

The characteristic polynomial equation of the WT power generation system which is given by (29). Comparing in (29) with (53), the coefficients are:

$$a_0 = 48,\ a_1 = -80,\ a_2 = 48,\ a_3 = -12, a_4 = 1$$

The sub-determinants are therefore, given by:

$$D_1 = a_1 = -80 \quad (57)$$

$$\begin{cases} D_2 = \begin{vmatrix} a_1 a_0 \\ a_3 a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3 \\ D_2 = \begin{vmatrix} a_1 a_0 \\ a_3 a_2 \end{vmatrix} = (-80)(48) - (48)(-12) = -3264 \end{cases} \quad (58)$$

$$D_3 = \begin{vmatrix} a_1 a_0 0 \\ a_3 a_2 a_1 \\ 0 0 a_3 \end{vmatrix} = a_3 D_2 = (-12)(-3264) = 39{,}168 \quad (59)$$

3. RESULTS AND DISCUSSIONS

For a Lyapunov function in variables $x_1, x_2, \ldots\ldots, x_n$ which can be put in matrix form $V(x) = x^T P x$, where $\boldsymbol{P}$ is a real symmetric matrix; necessary and sufficient conditions for $V(x)$ to be positive-definite are provided by Sylvester's criterion, which states that all the principal minors of P of order 1, 2, …….,n must be positive. That is:

$$p_{11} > 0, \begin{vmatrix} p_{11} p_{12} \\ p_{12} p_{22} \end{vmatrix} > 0, \begin{vmatrix} p_{11} p_{12} p_{13} \\ p_{12} p_{22} p_{23} \\ p_{13} p_{23} p_{33} \end{vmatrix} > 0, \ldots\ldots\ldots\ldots, |P| > 0$$

Determining the principal minors based on Sylvester's criterion, $p_{11} = -0.2083$. Thus, $p_{11} < 0$ does not meet the criteria. $\begin{vmatrix} p_{11} p_{12} \\ p_{12} p_{22} \end{vmatrix} = 0.04165$. Thus, $\begin{vmatrix} p_{11} p_{12} \\ p_{12} p_{22} \end{vmatrix} > 0$which meets the criteria.

$$\begin{vmatrix} p_{11} p_{12} p_{13} \\ p_{12} p_{22} p_{23} \\ p_{13} p_{23} p_{33} \end{vmatrix} = -0.00781. \text{ Thus, } \begin{vmatrix} p_{11} p_{12} p_{13} \\ p_{12} p_{22} p_{23} \\ p_{13} p_{23} p_{33} \end{vmatrix} < 0 \text{ does not meet the criteria.}$$

$$\begin{vmatrix} p_{11} p_{12} p_{13} p_{14} \\ p_{12} p_{22} p_{23} p_{24} \\ p_{13} p_{23} p_{33} p_{23} \\ p_{14} p_{24} p_{34} p_{44} \end{vmatrix} = 0.0013. \text{ Thus, } \begin{vmatrix} p_{11} p_{12} p_{13} p_{14} \\ p_{12} p_{22} p_{23} p_{24} \\ p_{13} p_{23} p_{33} p_{23} \\ p_{14} p_{24} p_{34} p_{44} \end{vmatrix} > 0 \text{ meets.}$$

Thus, by Sylvester's criterion, all the principal minors of P are not positive-definite and for that reason, the large-scale WT power system being considered is therefore, not asymptotically stable.

For a system to be Hurwitz stable, the determinants must meet the following criteria: *D1* > 0; *D2* > 0; and D3 > 0. The results obtained in the Hurwitz analysis are: $D_1 = -80$, $D_2 = -3264$ and $D_3 = 39{,}168$. From the results, only D_3 meets Hurwitz criteria which is an indication that the large-scale WT power system under consideration is also not Hurwitz stable. In the case of the matrix exponential analysis, the large-scale wind power system is stable if $e^{At} \to 0$ as $t \to 0$. All trajectories of e^{At} must converge to zero (0) as $t \to 0$. Thus, the obtained matrix A of the large-scale wind power system is stable if and only if all eigenvalues of matrix A have negative real part: $\Re\lambda_i < 0, i = 1,2, n$. As the $\lim_{t\to\infty} e^{At} \neq 0$, is an indication that the large-scale WT power system as well is not asymptotically stable which means mode response decay cannot be guaranteed for large, t. The results obtained through this modelling approach and analysis is less time consuming as compared to computer modelling and simulations to observe the results.

4. CONCLUSION

Different approaches namely Lyapunov, exponential matrix and Hurwitz were used to determine the stability of the large-scale renewable energy power grid. All the three approaches confirmed that the stochastic model is unstable. The assumptions made during the modelling stage can be revised to improve upon the model's stability; or a different modelling and analysis approach can be employed to ascertain the stability of such a power system. Ascertaining the actual behavior pattern will ensure that the appropriate control systems are designed for the stabilization of the WT power system based on the model. The primary contribution of this research is the development of a stochastic model for entirely renewable power grid which is envisaged to be implemented in the near future as result of dwindling fossil fuel reserves and environmental concerns. The stability of the grid was analyzed using three different approaches. All the three approaches used confirmed the hypothesis that the proposed large-scale renewable power grid might not be stable based on the modelling approach. It is noteworthy that this research did not factor battery storage system into the development of the model.

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