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New formulas generalized to the evaluations of solar irradiations captured on horizontal surfaces and optimal inclinations

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ABSTRACT

This work offers two significant contributions. The first concerns the proposal of a new formula for evaluating solar radiation on a horizontal plane in the sense of Joseph Fourrier's thermal equation. From which we deduce the characterization of solar radiation under overcast and almost overcast conditions. The second approach is dedicated to the calculation of solar irradiation captured on a fixed inclined surface. This consists of adding the expression of solar radiation coming from the horizontal plane with the overall balance of losses along the path of solar radiation. It appears that, contrary to the results of the models resulting from the Angstrom Prescott formula, the coefficients R= 0.9972, R^2= 0.9952, and MAPE= 0.061 for the Garoua data and R= 0.8849, R^2= 0.9407, and MAPE= 0.05, for the El Jadida data show that the results of the first proposed formula are well correlated with the measured values. Furthermore, using the optimal tilt angles, the second formula we proposed presents well-correlated results, such that: R= 0.9997, R^2= 0.9978, and MAE= 4.1470 for Garoua data and R= 0.9994, R^2= 0.9959, and MAE= 7.7742 for El Jadida data.

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1. INTRODUCTION

Africa on a global scale is an energy giant due to its enormous potential in diversified energy resources across the continent. However, Africa does not even consume more than 4% of the electrical energy produced in the world [1]. Many observers around the world are unanimous in observing that the continent's energy resources are underexploited by Africans, sometimes exported and often wasted during their exploitation. Thus, the current electricity production capacity in Africa is still very insufficient. Broadly speaking, the continent's energy challenges can be summarized into the following two main objectives [1]:

- Enable the development of strategies and techniques aimed at the efficient, optimal, and sustainable exploitation of energy resources.
- Constantly address Africa's energy deficit by relying primarily on the efficient use of its clean energy resources such as wind, geothermal, and solar.

In order to effectively and sustainably support projects linked to the solar sector, we will mainly focus in this work on proposing solutions aimed at the evaluation and prediction of the solar deposit with which the world and Africa in particular is richly endowed. To do this, two study sites will attract our attention for data collection in the town of Garoua in Cameroon and in the town of El Jadida in Morocco. The choice of these two regions of Africa is simply based on factors such as their climatic specificities, the availability of sunshine measurement data and accessibility to all the data for these regions. Other selection criteria concern the great ambitions and major projects that most African countries constantly harbor for the development of their solar energy sector. Geographically, Cameroon is a country located in central Africa at the bottom of the Gulf of Guinea which is part of the intertropical zone of the continent. It extends between 1°40' and 13°05' north latitude of the equator, and between 8°30' and 16°10' east longitude. Cameroon receives on average 5 to 6 kWh/m^2 of solar radiation [2]. While the Kingdom of Morocco in North Africa on the Atlantic and Mediterranean coast, is located at approximately 28°57' North latitude and 9°7' West longitude. The average solar radiation in Morocco is between 4.5 kWh/m^2 and 5.5 kWh/m^2 [1]. In practice, solar radiation is a key factor in the study and efficient and sustainable implementation of all possible solar projects, namely [2]:

- The development of solar energy systems such as power plants based on solar thermal collectors, and solar photovoltaic generators.
- The establishment of irrigation systems based on the calculation of potential evapotranspiration (ETP), to estimate the water needs of plants.
- The creation of meteorological or climatic centers.

Let us also recall that the values which characterize the global solar radiation in a given location can mainly be acquired by three classic means using [2]:

- Measuring instruments such as the pyranometer for global and diffuse measurements, and the pyrheliometer for direct measurements of solar radiation.
- Measurements by satellite images.
- Mathematical models based on the use of meteorological data and/or geographic coordinates of a chosen environment.

Furthermore, we encounter two main categories of model's characteristic of the solar irradiation which reaches the ground. The first category fundamentally concerns the determination of global solar radiation on a horizontal plane. The latter is the sum of the direct radiation having passed through the atmosphere and the diffuse radiation coming from all directions of the earth's roof scattered by particles, clouds, dust, and aerosols [3]. The second characteristic aims to evaluate the overall solar radiation on the inclined plane, the sum of which is composed of direct, diffuse and reflected radiation from the ground in the direction of the collection plates [3], [4]. In practice, the interest of solar irradiation models based on the inclinations of the collection surfaces makes it possible to quantify and predict the improvement in yield due to the optimal inclinations of photovoltaic panels, conventional solar stills (CSS), and reflectors [4]-[7]. In addition, the irradiation models when associated with other algorithms can be used for the determination and/or automatic adjustment or by the internet of things (IoT) of the optimal inclination angles of the surfaces of capture [4], [8]. Indeed, the determination of solar radiation to study meteorological phenomena and/or to meet energy needs constitutes the study of complex systems which depend on climatic, geographical, orographic, and cosmic factors. However, the modeling of complex systems remains a major challenge in the search for scientific and technical solutions to better analyze the dynamics of systems [9], [1]. As an example of the study of a complex system, Sharma *et al.* [10] proposes an approach for the long-term forecast of solar photovoltaic (SPV) power using a memory model long term (LSTM) with a Nadam optimizer. In order to correlate the power of the PV with the climate, the approach of Sharma *et al.* [10] takes into account parameters such as atmospheric temperature (AT), relative humidity (RH), insolation or solar radiation, and wind speed. This shows a good improvement in forecasting compared to autoregressive integrated moving average approaches and compared to models using optimizers such as RMSprop, Adam, and SGD. However, like all other sophisticated models, the approach of Sharma *et al.* presents as a major drawback the complexity due to the use of a high number of parameters which causes an increase in calculation time, a high risk of errors and delicate adjustments to the envisaged results. Moreover, a good study or good sizing of systems aimed at exploiting solar energy in a given region would only be possible if the solar radiation measurement data are easily exploitable, accessible or available, reliable, and available quantity [11]. However, we also note that this data is often rare for multiple reasons such as the long enough time to collect and process the measured data, the high costs of measuring devices and system maintenance, protection policies data is still practiced by certain states [12]. Hence the need to develop alternative formulas to save time in the production of forecast data, facilitate the rapid development of good specifications or technical specifications until their implementation, optimization or the practical validation of solar energy operating systems.

As a result, Chabane *et al.* [13], developed a theoretical method for calculating global solar radiation on a horizontal surface based on climatic conditions, parameters linked to pollution (CO, CO2, CH4, and O3) and water vapor. We can criticize the model of Chabane *et al.* [13] for its approximate results (high error rate), the rarity and the use of several calculation parameters and measurements which increase the complexity of using the model. In addition, Chabane *et al.* [14], propose yet another model to predict global solar radiation on a horizontal plane and on a plane of fixed inclinations. This comes from a linear regression based on the use of sun positioning equations, daily local time and five arbitrarily chosen inclinations. The validation of the model by Chabane *et al.* [14], presents a good correlation with the measured values and the values of the Perrin and Brichambaut model giving the R2 coefficients between 0.98% and 0.72% for different types of sky clarity. While Takilalte *et al.* [15], propose an empirical model limited to the calculation of normal direct solar irradiation (IDN). The latter is a function of the Perrin and Brichambaut radiation multiplied by the ratios of sunshine and solar radiation implying sky clarity indices. The results such as MAE= 11.05%, RSEI= 14.73%, and R^2= 0.87%, show that the proposed IDN can compete with classical models or based on intelligent algorithms [16].

Moreover, to calculate the global solar radiation on a horizontal plane, a large number of researchers around the world base themselves on the calculation model of Glover and Mc Couller of (1) [2], [4].

$$G = G_o(0.29 \cos L + 0.52 \frac{h}{H_0}) \quad (1)$$

Where, h: the duration of effective insolation or the time during a day during which the direct radiation actually reached the ground; H: the maximum possible duration of insolation; G: global solar radiation on a horizontal surface; G_0: extraterrestrial global solar radiation; and L: the latitude of the location.

The Glover regression coefficients for a study area in Kenya are 0.29 and 0.52. This model, as in (1) rather recommended by its authors for tropical regions, will be globalized too quickly and used by a good number of researchers throughout the world. Thus, according to the work of Angstrom-Prescott and al in 1969, this Glover model will become the model best adapted to the climatic specificities of the regions by rewriting it in the form of (2) [2], [17]-[19].

$$G = G_o(a + b \frac{h}{H_0}) \quad (2)$$

Where, a and b the constants which depend on the location. They are calculated using the regression analysis of the pairs of ratios G/G_0 and h/H_0 of the Angstrom relationship [19].

Let us also remember that the regression calculation used requires a good quantity of data measured in situ, in order to better appreciate the correlation between the calculated values and the measured data. Several other models for calculating solar radiation as a function of temperature are proposed in the work of Soulouknga *et al.* [18]. A comparative study between these different models for estimating global solar radiation as a function of temperature was carried out by Soulouknga *et al.* [18], it appears that the Sabbagh model presented in [18] is the most promising. This study was carried out using meteorological data over 26 years in the Abéché site in Chad, the best performances obtained with the Sabbagh model give the following statistical indicator values: R2= 0.706; RRMSE= 0.735%; and an acceptable error value, err= -3.704%. If the six models have the particularity of depending on temperature, and are also suitable for other specific fields of application, their performance nevertheless remains delicate and questionable with regard to the performance of the models resulting from (2). Thus, the list of works using the model of (2) is far from exhaustive. To stay within our study context, we will simply cite the following works:

- Ayangma *et al.* [2], proposed a correlation of (2) for the Garoua region, whose calculations of the Glover regression coefficients gave the values 0.30 and 0.43 for a quantity of data of sunshine and global radiation ranging from 1991 to 1993. It emerges from this analysis that the correlation and determination coefficients give the values 0.9875 and 0.9566 respectively.
- Rerhrhaye [20] and Zaimi [21], presented in their doctoral thesis work, a correlation of (2) whose calculations of Glover's regression coefficients gave the respective values around 0.33 and 0.42 for a quantity of sunshine and global radiation data ranging from 1991 to 1993 in the El Jadida region. It also emerges from their analyzes that the correlation and determination coefficients give approximately 0.80 and 0.78 respectively.
- Mirzabe *et al.* [22], developed 13 new model formats (including 05 daily formats and 08 monthly formats) to predict global solar radiation on the horizontal plane. The Angstrom-Prescott model is at the origin and inspired the development of all these different formats of polynomial models varying between the 1st and 4th order, proposed by [22]. Apart from the recently collected solar radiation and sunshine data (between 2008 and 2017) on 06 Iranian cities and 04 cities in other countries, the formats are adjusted by regression also using the input parameters such as: day number, declination angle, calculated hours of sunshine and degrees of cloud cover [22]. The results of the comparisons proved that the order 4 polynomial format model (ADM6) has superior performance to other developed models [22]. The statistical indicators of the ADM6 model associated with the thirteen cities studied give values for the correlation coefficients (R) between 0.9071 and 0.9692; and for the determination coefficients (R2) the values between 0.9963 and 0.9998. Furthermore, with regard to the evaluation of solar radiation on an inclined collection surface, the model most often encountered in the literature is that of Duffie *et al.* [23]. However, this model composed of

direct, diffuse and reflected radiation takes into account the use of charts and the calculations of a large number of geographical and climatic parameters of a considered environment. This makes the use of Duffie *et al.* model complex, particularly in terms of the risk of high errors, restrictive precision and calculation time [22], [24]. Despite the publications of several types of global model formats for predicting solar radiation, the modeling of complex systems presenting a better compromise between precision of results and simplicity or flexibility of the model envisaged still remains a very delicate and topical problem in the field of research [9], [1], [22]. It is clear that the performance of the solar radiation estimation models mentioned above can still be considerably improved.

To do this, the main objective of our study is to propose two new mathematical models that are more efficient, flexible, and optimal for predicting the solar energy potential on a horizontal plane and on an inclined collection surface. Specifically, we offer two formulas better suited to the climatic and orographic particularities of a place, guaranteeing a better reconciliation between the estimated values and those actually measured. Furthermore, the considered models rely on the strengths such as the use of fewer parameters, the reduction of time and errors in calculation and data processing, and the flexibility of the models to be usable in other domains. applications. The first proposed model will mainly focus on the determination of global solar irradiation on a horizontal plane, using the statistical processing of meteorological data of sunshine and solar irradiation for a period ranging from 1991 to 1993 in the regions of Garoua and from El Jadida. This new approach is based on Joseph Fourrier's formulation which deals with the propagation of heat by thermal conduction. Concretely, this consists of evaluating the average annual attenuation of solar radiation as it travels through the earth's atmosphere, in order to estimate the average maximum quantity of radiation that a solar collector placed on the ground can receive. While the second model that we propose for the evaluation of global solar radiation on any inclination plane, is obtained by associating the solar radiation of a horizontal plane with the balance of solar radiation lost on an inclination surface any. Thus, from the sunshine data of Garoua and El Jadida, the validation of our second proposed formula consists of studying the correlation of the results produced by the model of Duffie *et al.* [23] and the results obtained by this second proposed model.

This work is organized and presented as follows:

- The first part concerns the methodology, in which we recall the generalities on the notions of solar radiation on the ground, and the notions of propagation of heat by thermal conduction based on Joseph Fourier's heat equation. The latter is then transposed by analogy to the modeling of the loss of solar radiation through the atmosphere, in order to arrive at a new model for estimating global solar radiation on a horizontal plane. Consequently, we also deduce the formula for estimating the radiation lost through the atmosphere, while presenting the summary tables of the measured data and those calculated via the expressions of maximum possible duration of insolation and calculation of solar radiation extraterrestrial. On the one hand, a calculation reminder is presented for the determination of global solar irradiation on any plane based on the model of Duffie *et al.* and on the other hand, a detailed demonstration of the new solar irradiation calculation model on any tilting surface is also exposed.
- The second part is devoted to the validation studies of the two new models proposed for the calculations of solar radiation on a horizontal plane and on any plane of inclination. This phase concerns the results and the discussions, relating to the comparison of the performances between the proposed models, the measured values and the old models.
- The third part is devoted to the conclusion.

2. METHOD

2.1. Methodologies for implementing a new model for calculating global irradiation on a horizontal plane

The methodological approach which allowed us to propose two new formulas for forecasting global solar radiation on a horizontal plane and on an inclined plane is described in two main steps as follow:

- The first step concerns recalling generalities on the notions of solar radiation which reaches us on earth, and the notions of the propagation of heat by thermal conduction of Joseph Fourier. This notion of thermal conduction is transposed by analogy to the modeling of the loss of solar radiation through the atmosphere, in order to arrive at a new model for estimating global solar radiation on a horizontal plane. From this modeling, the formula for estimating the radiation lost through the atmosphere which can characterize solar radiation under an overcast sky and under an almost overcast sky will also be deduced. For the numerical and qualitative validation, we will recall the expressions of the parameters such as the maximum possible insolation duration H, the solar radiation outside the atmosphere γo, and the statistical performance indicators R, R^2, and MAPE.
- The second stage is dedicated to the presentation of a new approach for determining global solar radiation on an inclined plane. The equations which describe the apparent movement of the sun and the search for the optimal capture angle are presented, in order to be used in the calculations of the global solar irradiation on

an inclined plane of the Duffie *et al*. model. The demonstration presented last to determine the solar radiation proposed on an inclined plane, consists of associating the solar radiation on a horizontal plane with the overall loss balance along the path of the solar radiation.

2.1.1. Fundamentals of solar radiation received on earth

Figure 1 shows that solar radiation is electromagnetic radiation essentially composed of: visible light; of wavelength between 400 nm and 800 nm; infrared (IR) radiation of wavelength less than 400 nm; ultraviolet (UV) radiation with a wavelength greater than 800 nm. On Earth, the atmosphere (via carbon dioxide, ozone, and water vapor) largely absorbs IR and UV and a little visible light. Thus, the greater the thickness of the atmosphere, the lower the amount of solar energy received on the ground [25], [3]. Indeed, when we move towards the poles, the sun's rays are more inclined, and the amount of energy received on the ground occupies a large surface. Consequently, the solar radiation per unit area received decreases from the equator towards the poles; this is at the origin of the phenomenon of the seasons, when we also take into account the inclination of the axis of the Earth. Solar energy is also reduced by the alternation of days and nights, by cloud cover (this reduces solar energy by about 50%), and by seasonal variations [25], [26]. The determination of the solar deposit will consist for us of modeling and evaluating or characterizing all of the three main factors of attenuation of the solar radiation generated, using a new mathematical approach based on the interpretation of the conduction phenomenon thermal in the sense of Joseph Fourier.

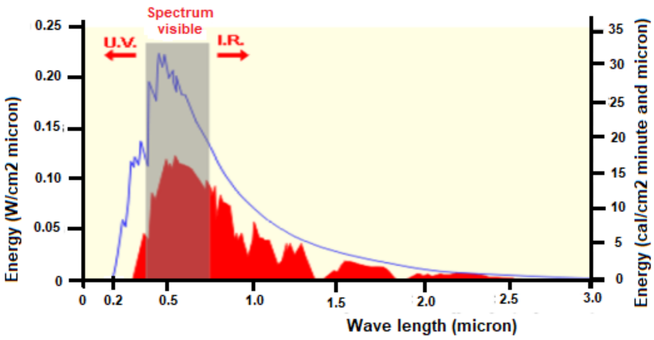

Figure 1. Spectral distribution of solar radiation

2.1.2. General information on the propagation of heat by thermal conduction

The propagation of heat by thermal conduction in a medium corresponds to an internal energy transfer due to the interactions between the particles which constitute a thermodynamic system (example: shocks of molecules in gases, and vibration in crystalline solids). The transfer of energy is present in all bodies whatever their states (solid, liquid, or gas). For example, one can directly experience a mode of transfer by conduction, by holding a metal bar in the hand, and putting the other end in contact with a flame. After a while, you have to let go of the bar to avoid getting burned. The bar experiment suggests that the flow of heat which propagates by conduction in matter is linked to variations in temperature. Hence, Joseph Fourier in 1822 found from this experiment that there is a relationship between the heat flux density and the spectral temperature variation, as follows (Fourier's Law) [26], [27].

$$\vec{q} = -\lambda \overrightarrow{grad} T \quad (3)$$

Where, T: temperature (°k), λ: thermal conductivity of the medium, and $||\vec{q}||$: heat flux density vector (w/m.k).

a) Planar geometry of the illustrative example of the determination of the temperature field in a one-dimensional (1D) wall

We consider a plane wall (see Figure 2) whose lateral dimensions are much greater than its thickness e (with negligible edge effects) and subjected on its faces to temperature variations with uniform boundary conditions. The objective pursued is twofold, that of determining the temperature field in the wall, as well as the heat flux passing through it. That is, the direction orthogonal to the wall (see Figure 2), with the assumption that the temperature depends only on the abscissa x. We say that the problem is one-dimensional (or 1D) in x. Under these conditions, the equation that gives the temperature field becomes very simple. The heat equation reduces to the Poisson equation as (4) [26], [27].

$$\Delta T + \frac{W}{\lambda} = 0 \quad (4)$$

If moreover the work exchanged is zero that is to say if the transfer of energy is reduced to the transfer of heat, we obtain the Laplace equation $\Delta T= 0$; and in Cartesian coordinates we have (5).

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0 \tag{5}$$

We can write $\frac{\partial^2 T}{\partial x^2}=0,\ \frac{\partial^2 T}{\partial y^2}=0,\ \frac{\partial^2 T}{\partial z^2}=0$

For T_0 the temperature at x= 0 and T_1 the temperature at x= e, the temperature field in the wall is written as (6).

$$T(x)=T_0+\frac{(T_1-T_0)}{e}x \tag{6}$$

The temperature in the wall varies linearly with x. The flux density at a point on the wall is given by Fourier's law, as in (7).

$$\vec{q}=-\lambda\frac{\partial T}{\partial x}\vec{x}=-\lambda\frac{(T_1-T_0)}{e}\vec{x} \tag{7}$$

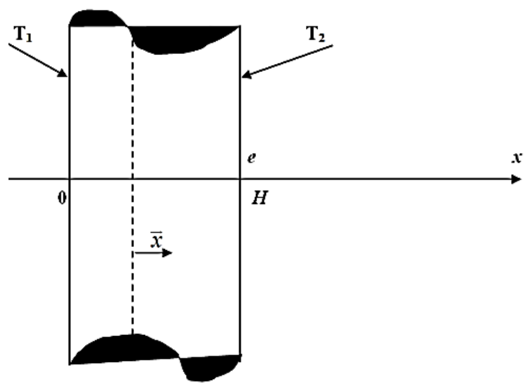

Figure 2. Plane wall of thickness (e)

b) Modelling of the solar deposit based on the interpretation of the loss of solar radiation through the atmosphere

By analogy to the determination of the temperature field in a wall as in Figure 2, we arrive at the formulation of a model for evaluating the loss of solar radiation through the atmosphere illustrated in Figure 3. The description of Figure 3 shows us that, the losses of sunshine along the atmospheric path naturally depend on the cloud cover and the variations of seasons, days and nights. In addition, the following elements of boundary conditions can be noted:

- The average solar radiation outside the atmosphere is γ_0 (Wh/m^2).
- The average solar radiation that can actually be captured during the day is γ_m (Wh/m^2).
- The h and H are respectively the effective and maximum possible insolation durations (hours).

Therefore, the mathematical formulation of the model proposed in Figure 3 can be written as (8).

$$\frac{\partial^2 \gamma_p}{\partial h^2}=0\rightarrow\frac{\partial \gamma_p}{\partial h}=A_1 \tag{8}$$

Where, A_1 is a constant and γ_p is the loss of solar radiation through the atmosphere. The resolution of (8) with partial derivatives (PDE) therefore leads us to write (9).

$$\gamma_p(h)=A_1 h+A_2 \tag{9}$$

Where, A_1 and A_2 the constants to be determined based on the boundary conditions.

According to Figure 3, for h= 0 we can write γ_p (h= 0) = γ_0 = A_2, therefore (9) will be written as (10).

$$\gamma_p(h)=A_1 h+\gamma_0 \tag{10}$$

The physical interpretation that corresponds to the time interval where h= 0 in the Figure 3, is that no solar radiation is yet detected on the ground. We can find ourselves at this moment (h= 0) in a situation of total occultation of the sun, or during the period of the night before sunrise and after the apparent sunset. In other words, when no solar irradiance is detected on the ground, we can write γ_p (h= 0) = 0 = γ_0. Consequently, the (10) can be rewritten based on the physical interpretation as (11).

$$\gamma_p(h) = A_1 h \tag{11}$$

Substituting (for h = H) in (10), we determine the (12) of A_1.

$$A_1 = \frac{(\gamma_m - \gamma_0)}{H} \tag{12}$$

To have a correct interpretation of our proposed model or physical phenomenon, we replace (12) in (11), and we will end up with a new equation for the loss of solar radiation through the atmosphere as (13).

$$\gamma_p(h) = \frac{(\gamma_m - \gamma_0)}{H} h \tag{13}$$

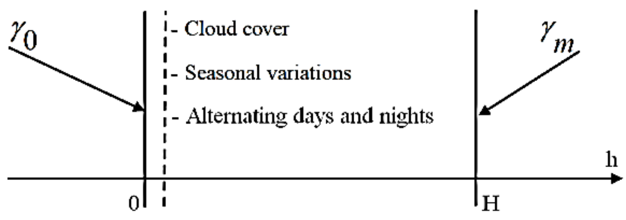

Figure 3. Modeling the path of solar radiation through the atmosphere

c) New model for predicting global solar radiation on a horizontal plane

The prediction of global solar radiation based on the loss equation (13) consists of a least-squares regression analysis using measured and calculated data. To do this, consider rewriting (13) as (14).

$$\gamma_{mi} = \gamma_{0i} + \gamma_p \frac{H_i}{h_i} \tag{14}$$

By identifying (14) with an equation function of the plane, we can write (15).

$$Z_i = f(x_i,\ y_i) = ax_i + by_i \tag{15}$$

Where, γ_{mi}= Z_i; γ_{0i}= y_i; H_i /h_i= x_i; a and b the regression coefficients to be determined.

The adjustment of (15) by the method of least squares consists in minimizing the function S (a, b) of the square of the error (S (a, b) = e^2_i =| (ax_i+by_i) – Z_i|^2), as follows:

$$\begin{cases} \frac{\partial S(a,b)}{\partial a} = 0 \\ \frac{\partial S(a,b)}{\partial a} = 0 \end{cases} \rightarrow \begin{cases} 2\sum(ax_i^2 + by_i x_i - Z_i x_i) = 0 \\ 2\sum(ax_i y_i + by_i^2 - Z_i y_i) = 0 \end{cases} \tag{16}$$

Let's pose:

$$\begin{cases} S_{xx} = \sum x_i^2\,;\ \ S_y = \sum y_i;\ S_{zx} = \sum Z_i x_i \\ S_{xy} = \sum x_i y_i\,;\ \ S_{yy} = \sum y_i^2;\ S_{zy} = \sum Z_i y_i \end{cases} \tag{17}$$

substituting the elements of (17) into (16), we get (18).

$$\begin{cases} aS_{xx} + bS_{xy} = S_{zx} \\ aS_{xy} + bS_{yy} = S_{zy} \end{cases} \tag{18}$$

By rearranging (18), we deduce as (19) for a and b.

$$\begin{cases} b = \frac{S_{zx}S_{xy} - S_{zy}S_{xx}}{S_{yx}S_{yx} - S_{yy}S_{xx}} \\ a = \frac{S_{zy} - bS_{yy}}{S_{yx}} \end{cases} \quad (19)$$

Therefore, the new model proposed for the prediction of global solar radiation on a horizontal plane can be written according to the expressions of (19) as (20).

$$\gamma_{mi} = b\gamma_{0i} + a\frac{H_i}{h_i} \quad (20)$$

Furthermore, the expression for the solar radiation retained or dissipated in the atmosphere can be deduced from (13) and/or the (21).

$$\gamma_{Di} = \gamma_{0i} - \gamma_{mi} \quad (21)$$

With, γ_{Di} (Wh/m^2): average solar radiation lost or retained in the atmosphere at a time when $h_i = H_i$.

2.1.3. Expressions of the calculated parameters of the global solar radiation model

The parameters h and γ_m are obtained by measurements and in the form of data samples, while the other parameters of the proposed model are determined by calculation as follows:

a) Maximum possible duration of insolation H

The maximum possible duration of insolation is defined as the time interval between sunrise and sunset. It was calculated by (22) [2], [28].

$$H = (TLc - TL\ell) = 2/15cos^{-1}(-\tan L \tan\ \delta) \quad (22)$$

Where, TLc and TLℓ respectively the time at sunrise and the time at sunset and δ the declination of the sun.

b) Calculation of solar radiation entering the atmosphere γo

For an average earth-sun distance, the illumination or power received by a surface normal to the rays of the sun, located at the entrance to the atmosphere is estimated at a so-called solar constant value equal to 1367 w/m^2. The average variation during the year of this solar constant corresponds to (23) [19], [28].

$$\gamma_{0n} = 1367[1 + 0.034\cos(0.986(j - 3))] \quad (23)$$

With j is the number of the day of the year. The instantaneous values of the extraterrestrial solar irradiation received by a horizontal surface during a day between the instant of sunrise and that of the apparent sunset are obtained by (24) [19], [28].

$$\gamma_0 = \int_{t\ell}^{tc} \gamma_{0n}\sin(h_s)dt = \frac{24}{\pi}\sin L \sin\delta\left(\frac{\pi}{180}\omega_c - \tan\omega_c\right) \quad (24)$$

Where, h_s: he height of the sun and ω_c: the hour angle at sunset.

2.1.4. Statistical indicators for evaluating the performance of the mathematical prediction model

There are several types of statistical indicators used for performance evaluation and validation of the mathematical prediction model. The main indicators we used are as follows:

a) Correlation coefficient R

The correlation coefficient noted R (for R very close to the value 1), is a performance criterion which informs us that the values predicted by the calculation model and those obtained by measurements go in the same direction, and are very close to each other each other. In addition, this also justifies the good choice of having used regression analysis in the sense of least squares. The determination of the correlation coefficient is as (25) [23], [28].

$$R = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(X_m^i - \overline{X_m^i}\right)\left(X_{cal}^i - \overline{X_{cal}^i}\right)}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_m^i - \overline{X_m^i}\right)}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_{cal}^i - \overline{X_{cal}^i}\right)}} \quad (25)$$

b) Coefficient of determination

The coefficient of determination noted R^2 (for $0 < R^2 < 1$), is a performance criterion which informs us about the evaluation of the dispersion of the values measured around the curve of the prediction or regression model. Furthermore, the coefficient of determination quantifies the degree of fit of the data to a regression model, and is calculated as (26) [20], [28].

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(X_m^i - X_{cal}^i\right)^2}{\sum_{i=1}^{N}\left(X_m^i - \overline{X_{cal}^i}\right)^2} \quad (26)$$

c) Mean absolute error percentage (MAPE)

The mean absolute percentage error (MAPE) evaluates the difference between the values measured and calculated by prediction models. It is determined by (27) [23], [28].

$$MAPE = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{X_m^i - X_{cal}^i}{X_m^i}\right| \quad (27)$$

2.2. Methods for implementing a new model for calculating global irradiation on any plane of inclination

The theoretical coordinates of the apparent movement of the sun and the terrestrial geographic coordinates are used to determine the optimal inclination angles which will be used in the new calculation of the global insolation on a fixed and inclined capture surface.

2.2.1. Apparent sun positions

The apparent motion of the sun seen from the earth is characterized by parameters like [29]-[31]. The hour angle, defined by (28).

$$\omega = 15°(TS - 12) \quad (28)$$

Where, TS represents solar time. For TS=0, we say that the height of the sun is at its maximum (at the zenith). The height of the sun (h_s) can then be deduced from (29).

$$\sin h_s = \sin L \sin \delta + \cos \delta \cos L \cos \omega \quad (29)$$

With, δ the declination of the sun, which varies throughout the year between the angles -23.45° and +23.45° [20], [3]. The expression for the declination as in (30).

$$\delta = 23.45 \sin[0.980(j + 284)] \quad (30)$$

The calculation of the azimuth (az) of the sun is as (31).

$$\sin az = \frac{\cos \delta \sin \omega}{\cos h_s} \quad (31)$$

2.2.2. Determination of the optimal orientation of the surfaces for capturing sunlight

In practice, there are mainly two techniques for optimizing the sun collection surfaces, the first of which concerns the determination of the various optimum angles of fixed positioning of the collection surfaces; while the second technique is based on the development of one-axis or two-axis automatic tracker methods [25], [30]-[33]. However, the technique of fixed optimal positioning is necessary and sufficient for the validation study of the new model for calculating global insolation that we are considering and proposing in this paper. To do this, consider the different positions of inclination of the capture surfaces in Figure 4. The latter shows that, the search for the optimal angle of inclination according to the latitude and the declination of the sun can write as (32) [25], [28].

$$Inc_{opt} = L \pm \delta \quad (32)$$

The optimal capture position sought can be denoted by the pair of coordinates (Inc_{opt}, y_a); where ya represent the orientation angle (y_a) of the receiver plane with respect to the south direction, and equals $y_a = 0$ oriented due south for sensors located in the terrestrial regions of the northern hemisphere and vice versa [25], [30], [32], [33]. Consequently, from the representations of Figure 4 for the regions of Garoua and El

Jadida, one can seek the optimal annual fixed inclination among the equation of the following pairs of coordinates, as in (33).

$$\left(Inc_{opt}, ya = 0\right) = (L, 0);\ \left(L \pm \frac{23.45}{2}, 0\right); \left(L \pm \frac{23.45}{4}, 0\right) \quad (33)$$

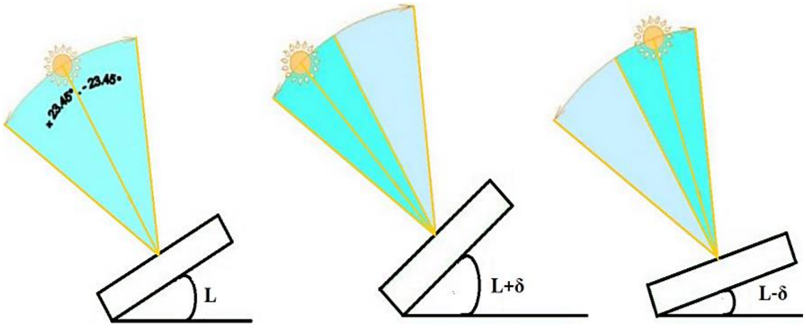

Figure 4. Finding the optimal orientation of solar collectors based on the latitude and declination of the sun

2.2.3. Evaluation of solar radiation on a flat surface of any inclination

The solar radiation on a plane surface of any inclination $G_{inc}(Inc)$, is broken down into direct radiation $S(Inc)$, diffuse radiation $D(Inc)$, and reflected radiation $R(Inc)$ by the ground, in order to be received by the oriented surface. The expression of this global solar radiation on a surface of any orientation proposed by Duffie *et al.* is [3], [30]-[35]:

$$G_{inc}(Inc) = S(Inc) + D(Inc) + R(Inc) \quad (34)$$

with:

$$S(Inc) = S_d(\cos h_s \sin Inc \cos(az - ya) + \sin h_s \cos Inc) \quad (35)$$

S_d represents the normal or direct flux density on the plane perpendicular to the solar rays.

$$D(Inc) = D_{di}(1 + \cos Inc)/2 \quad (36)$$

D_{di} represents the diffuse flux density on the horizontal plane.

$$R(Inc) = G\rho(1 + \cos Inc)/2 \quad (37)$$

G represents the global solar irradiation on the horizontal plane and ρ is the reflectance were called albedo.

Theoretically, let us recall that the calculation of the global solar radiation on a plane surface of any inclination depends strongly on the determination of the parameters which characterize the apparent movement of the sun and on the calculation of the angles of optimal inclinations of the surfaces of captures installed in a place given. Moreover, according to the formulas proposed by Duffie *et al.* and by Liu and Jordan in [22], the solar radiation on an optimally tilted capture surface also depends on other parameters such as the albedo (ρ), the haze factor of Linke, monthly average clarity index (Kt), and monthly diffuse fraction (S_d/G) [22], [30], [35]. Therefore, the formula we propose for the evaluation of solar radiation on an optimal collection surface comes from (35), which can also be written in (38).

$$S(Inc) = S_d \cos i \quad (38)$$

Cos (i): represents the angle of incidence of radiation on a plane measured from the normal.

Considering that the installation of a solar collection panel is installed in full south, we can say that the azimuth angle (az) of the sun, and the orientation angle (y_a) of the receiving plane by relative to a southerly direction, are zero. Therefore, the angle of incidence as a function of the angle of inclination (Inc) of the solar receiver and the angle of the height of the sun can be written as (39).

$$\cos i = \sin(h_s + Inc) \quad (39)$$

From (38), it can be seen that the angle of incidence also acts as a factor reducing the effectiveness of direct radiation. Therefore, the amount of effectively lost direct radiation (Q_{dp}) can be given as (40).

$$Q_{dp} = S_d(1 - \sin(h_s + Inc)) \quad (40)$$

The direct flux density on a surface normal to the rays of the sun is given as (41) [13].

$$S_d = \propto G \quad (41)$$

For a clarity index of 0.17<Kt<0.75, the monthly diffuse fraction can be determined as (42) [22], [34], [35].

$$\propto = 1.88 - 2.272Kt + 9.473Kt^2 - 21.865Kt^3 + 14.648Kt^4 \quad (42)$$

By subtracting (40) from (13), we can deduce the amount of solar radiation that would actually be captured by a solar receiver at optimal tilt, whose expression is as (43).

$$Q_c = \gamma_p - Q_{dp} \quad (43)$$

By adding (43) with the global solar radiation measured or calculated on the horizontal plane (G_m or γ_m), we arrive at the proposal of the formula for evaluating solar radiation on a collection surface at any inclination, as (44).

$$\gamma_{Inc}(Inc) = \gamma_m + \left[\gamma_p - \alpha\gamma_m(1 - \sin(h_s + Inc))\right] \quad (44)$$

3. RESULTS AND DISCUSSION

The validation of the proposed formulas is based not only on the exploitation of sunshine and solar irradiation data collected between the years 1991 and 1993 in the regions of Garoua in Cameroon and El Jadida in Morocco. But also, on the analysis of the statistical indicator results of the curves obtained from the models proposed for the calculations of global insolations on the horizontal plane and on the inclined plane. All of this work was carried out using MATLAB software.

3.1. Validation of the new model proposed for the calculation of solar radiation on a horizontal plane

In order to validate our new approach for predicting global solar radiation on a horizontal plane, we first use two data samples of measurements of effective insolation and solar irradiation (h_i, γ_{mi}) for a period ranging from from 1991 to 1993 in the towns of Garoua and El Jadida. Secondly, from the measured data and the calculation formulas, the values of the maximum possible insolation and the solar irradiation outside the atmosphere ($H_{i,\ oi}$) are deduced. Based on the samples of data collected (between 1991 and 1993) and the calculated values, an analysis of the adjustments of the model of (20) proposed was made by the method of least squares, in order to determine the values of the regression coefficients a and b. The results of these adjustments obtained give us the pairs of coefficients (a= -1954.5 and b= 0.8933) and (a= -542.4 and b= 0.717) respectively for the Garoua and El Jadida data associated with the prediction models solar radiation. As a result, the equations of the new model for predicting global solar radiation on a horizontal plane in Garoua and El Jadida can respectively be written in the following model forms:

$$\gamma_{mi} = 0.8933\gamma_{0i} - 1954.5\frac{H_i}{h_i} \quad (45)$$

$$\gamma_{mi} = 0.717\gamma_{0i} - 542.4\frac{H_i}{h_i} \quad (46)$$

3.2. Estimation of the solar resource of the cities of Garoua and El Jadida

The statistical processing of the data collected and calculated for each day number of the month allows us to determine the monthly average values of the quantities h, H_o, G_o, G_m, and γ_m as summarized in Tables 1 and 2 of the cities of Garoua and d 'El Jadida respectively. To prove the effectiveness of our approach based on the data in Tables 1 and 2, we represent Figures 5 and 6 to clearly illustrate the fact that the global solar radiation curves obtained from the proposed models (γ_m (Wh/m^2)) are very close to the curves of the average monthly values measured (G_m (Wh/m^2) [2]). Furthermore, to better enrich the interpretation of the results of Figures 5 and 6, we still use the values of Tables 1 and 2 in the numerical applications of (25)-(27) of the statistical performance indicators.

Table 1. Parameters of sunshine and monthly average global irradiation in Garoua

Month	Jan	Feb	Mar	April	May	Jun	July	Aug	Sept	Oct	Nov	Dec
h (hour)	282	270	273	244	257	221	193	188	195	252	288	293
H_o (hour)	364	331	371	364	380	369	380	377	361	368	353	363
G_o (Wh /m^2)	8885	9553	10169	10397	10254	10081	10113	10237	10121	9604	8943	8612
Gm (Wh/m^2)	5628	6207	6262	6113	6063	5621	5239	5269	5387	5712	5827	5592
γ_m (Wh/m^2)	5418	6130	6425	6370	6275	5744	5180	5231	5423	5728	5588	5300

Table 2. Parameters of sunshine and monthly average global irradiation in El Jadida

Month	Jan	Feb	Mar	April	May	Jun	July	Aug	Sept	Oct	Nov	Dec
h (hour)	208	185	211	261	310	294	267	279	264	217	231	183
H_o (hour)	313	302	366	384	425	426	434	409	366	344	309	304
G_o (Wh /m^2)	5384	6682	8427	10066	11113	11497	11265	10400	8947	7175	5662	4977
Gm (Wh/m^2)	3100	3800	4900	6500	7100	7500	7100	6500	5800	4200	3600	2600
γ_m (Wh/m^2)	3046	3978	5230	6406	7157	7433	7266	6646	5603	4331	3250	2754

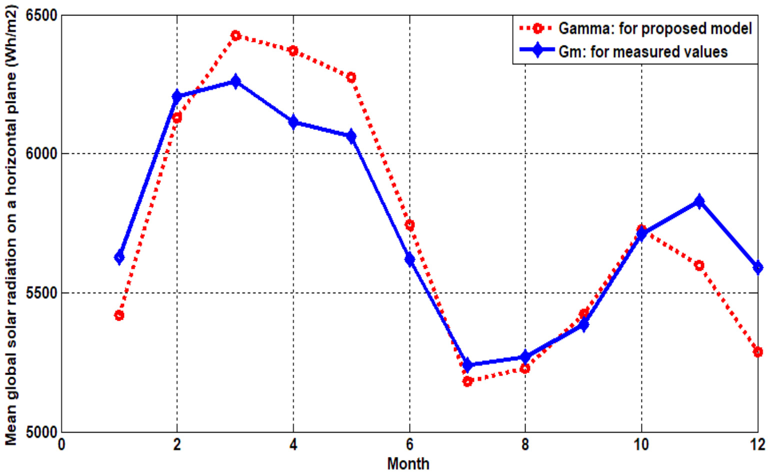

Figure 5. Monthly average global irradiations in Garoua

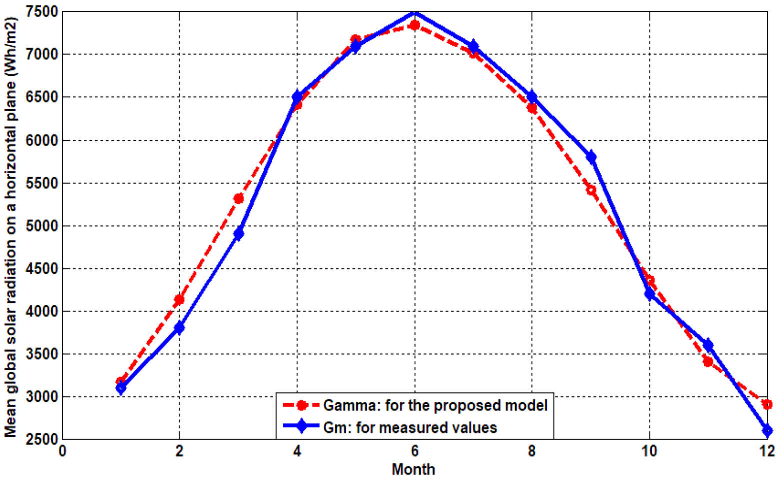

Figure 6. Monthly average global irradiations in El Jadida

Consequently, we obtain the comparative table, as in Table 3, which clearly shows that the values of the statistical indicators of the correlated models proposed for the cities of Garoua and El Jadida, present better performances compared to the correlated Angstrom-Prescott models in [2], [20], [21]. In addition, the results of the coefficients: R= 0.9972, R^2= 0.9952, and MAPE= 0.061 of the models proposed for Garoua and R= 0.8849, R^2= 0.9407, and MAPE= 0.05 of the models proposed for El Jadida; further confirm the fact that the values of solar radiation of the horizontal plane of the proposed models are strongly correlated with the values resulting from the measurements. In sum, the proposed model presents better performances compared to models derived from the Angstrom-Prescott formula in [2], [20], [21] and compared to other new model formats mentioned in [18], [22], particularly in terms of precision and flexibility (flexibility and simplicity).

Table 3. Comparative table of models for evaluating global solar radiation on a horizontal plane

Regions	Correlated Models of Each Region	Coefficients R, R^2, and Mape
City of Garoua	(Angstrom-Prescott model [2]) $G_i = G_{0i}\left(0.30 + 0.43\frac{h_i}{H_i}\right)$	R= 0.9875 R^2= 0.7920
	(Proposed model) $\gamma_{mi} = 0.8933\gamma_{0i} - 1954.5\frac{H_i}{h_i}$	R= 0.9982 R^2= 0.9952 MAPE= 0.0613
City of Jadida	(Angstrom-Prescott model [21]) $G_i = G_{0i}\left(0.32 + 0.44\frac{h_i}{H_i}\right)$	R= 0.92 R^2= 0.7788
	(Proposed model) $\gamma_{mi} = 0.7176\gamma_{0i} - 542.4\frac{H_i}{h_i}$	R= 0.9407 R^2= 0.8849 MAPE= 0.0782

3.3. Estimation of the loss of global solar radiation through the atmosphere on the cities of Garoua and El Jadida

The other great advantage of the approach that we propose in this article is based on the development of a new formulation of the dispersion of solar radiation through the atmosphere, defined according to the expression of (13). In other words, the (13) would be highly recommended if we would like to know or estimate the amount of solar radiation retained in the atmosphere during an effective sunshine duration and in a given region. Moreover, when we consider hi=Hi in (13), we can write the expression of the radiation retained in the atmosphere from the measured values as (47).

$$\gamma_{Di} = \gamma_{0i} - G_{mi} \tag{47}$$

Whereas, the radiation retained in the atmosphere from the values calculated by the proposed model can be written as (48).

$$\gamma'_{Di} = \gamma_{0i} - \gamma_{mi} \tag{48}$$

Using the values from Tables 1 and 2 in (47) and (48), we determine the monthly average quantities of global solar radiation dissipated through the atmosphere for the regions of Garoua and El Jadida respectively recorded in the Tables 4 and 5. The data in Tables 4 and 5 were used to draw the curves in Figures 7 and 8 for the towns of Garoua and El Jadida respectively, in order to clearly illustrate the correlations from which the values calculated dissipated radiation (γ_{Di} (kWh/m^2)) are very close to the measured dissipated radiation values (γ'_{Di} (Wh/m^2)). Furthermore, according to the work of Zaimi [21], we realize that the solar radiation curves of El Jadida for an overcast sky and an almost overcast sky give respectively from January to December the couples following values (in kWh/m^2) [21]: (1.4, 2.2), (1.8, 2.9), (2.4, 3.5), (3.6, 3.4), (4.2, 4.1), (4.4, 5.1), (2.4, 5.5), (3.6, 4.5), (2.1, 3.3), (1.7, 3), (1.7, 2.1), and (1.1, 2). These pairs of values are close to the pairs of values (γ_p, γ'_{Di}) in Table 5 and the curves in Figure 8. As a result, we can say that the radiation diffusion formula proposed in (13), allows us to determine the global solar radiation for an overcast sky and for an almost overcast sky. This allows us to assimilate γ_p to the quantity of solar radiation actually dissipated under overcast skies, while γ_{Di} will be assimilated to the quantity of solar radiation dissipated under almost overcast skies. In addition, the advantages of (13) take into account the duration of sunshine in a given region, and whatever the nature of climatic obstacles during a period considered.

Table 4. Average monthly radiation dissipated in the atmosphere on a horizontal plane in Garoua

Month	Jan	Feb	Mar	April	May	Jun	July	Aug	Sept	Oct	Nov	Dec
γ_{Di} (kWh/m^2)	3257	3346	3907	4284	4191	4460	4874	4968	4734	3892	3116	3020
γ'_{Di} (Wh/m^2)	3467	3423	3744	4027	3979	4337	4933	5006	4698	3876	3355	3312
γ_p (Wh/m^2)	2530	2720	2870	2869	2840	2672	2471	2481	2557	2668	2547	2453

Table 5. Average monthly radiation dissipated in the atmosphere on a horizontal plane in El Jadida

Month	Jan	Feb	Mar	April	May	Jun	July	Aug	Sept	Oct	Nov	Dec
γ_{Di} (Wh/m^2)	2284	2882	3525	3566	4013	3997	4165	3900	3147	2975	2062	2377
γ'_{Di} (Wh/m^2)	2338	2704	3197	3660	3956	4064	3999	3754	3344	2844	2412	2223
γ_p (Wh/m^2)	1515	1712	2033	2424	2930	2800	2560	2660	2270	1876	1542	1431

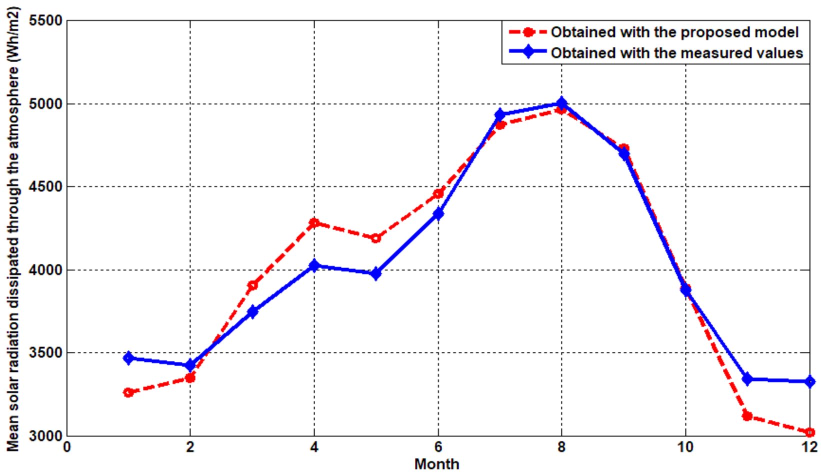

Figure 7. Monthly mean global solar irradiance dissipated in the atmosphere in the Garoua region

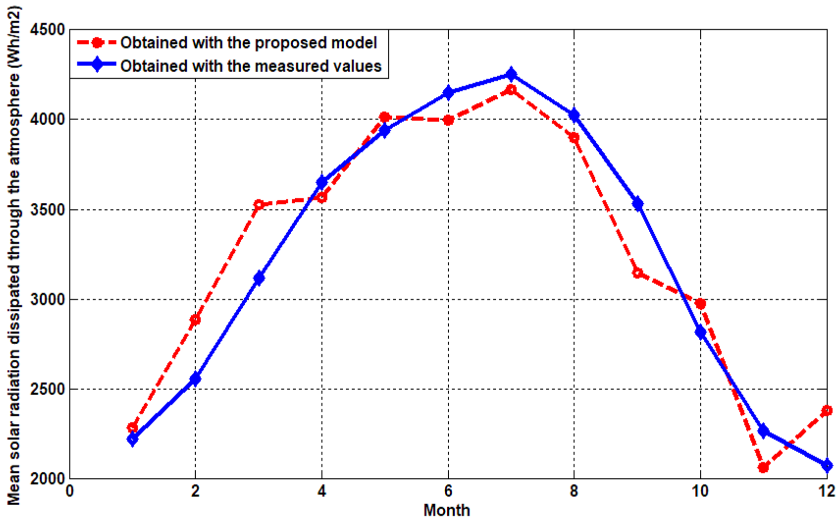

Figure 8. Monthly mean global solar irradiance dissipated in the atmosphere in the El Jadida region

3.4. Validation of the new solar radiation calculation model on any plane

The validation approach of the proposed model consists of evaluating the effectiveness of each pair of coordinates of (33) through the models of (44) and (34), and of deducing the optimal pairs of fixed annual inclinations for the two study regions Garoua and El Jadida. Next, we determine the different statistical performance indicators which will allow us to conclude that there is a strong correlation between the results obtained with the old model of (34) and the results of the new model of (44) proposed. Consequently, Tables 6 and 7 respectively give us the monthly average solar irradiance on a plane of fixed annual inclinations in the regions located in Garoua and El Jadida. Tables 6 and 7 contain the average luminosity index (α) for each month of the year, and the inclination angles (Inc) chosen from (33). Ginc represents the monthly average solar radiation of the Duffie *et al.* [23] model, while γinc is the monthly average solar radiation of the proposed model. The annual sum of average monthly radiation for each region is denoted GSA. Unlike the other angles, the optimal fixed inclinations of 27° and 40° respectively in Tables 6 and 7 of Garoua and El Jadida, make it possible to collect the maximum possible solar radiation throughout the year.

The plots of the histogram curves of annual sums of monthly average radiation (G_{SA}) illustrated by Figures 9 and 10, for which we observe a maximum annual average illumination and an annual fixed optimal inclination are respectively (G_{inc}= 95 kWh/m^2 and γ_{inc}= 101 kWh/m^2; Inc= 27°) for the Garoua region and (G_{inc}= 85 kWh/m^2 and γ_{inc}= 92 kWh/m^2; Inc= 40°) for the El Jadida region. Consequently, the two models studied allow us to identify the same optimal tilt angle for the same given region. However, the formula we propose effectively covers the maximum expected values faster than the model of Duffie *et al*.

Table 6. Average values of global solar irradiance on an annual fixed orientation plane in Garoua

Average values of global solar illuminance (Kwh/m^2) on a fixed annual orientation surface													
Inc (°)	Jan	Feb	Mar	Apl	May	Jun	July	Aug	Sept	Oct	Nov	Dec	G_{SA}
3.4° G_{inc}	6.9539	8.1600	7.6686	7.4900	7.4240	6.9096	6.4652	6.4694	6.6222	7.5557	7.1708	6.9094	85.7988
γ_{inc}	7.7873	8.8151	8.2258	8.6668	8.5904	7.9770	7.3674	7.4246	7.6099	8.0247	8.0070	7.6767	96.1727
9.3° G_{inc}	7.2165	8.3106	7.9018	7.9923	7.6273	7.1048	6.6858	6.6817	6.8629	7.7997	7.4338	7.1703	88.7875
γ_{inc}	7.9262	8.8730	8.9343	8.7859	8.7085	8.0939	7.4942	7.5483	7.7378	8.1316	8.1473	8.8146	99.1956
15.2°G_{in}	7.4181	8.4065	8.0728	8.0169	7.7768	7.2500	6.8560	6.8436	7.0456	7.9977	7.6338	7.4707	90.7885
γ_{inc}	8.0335	8.9102	9.0288	8.8790	8.8008	8.1861	7.5941	7.6447	7.8363	8.2698	8.2549	7.9213	99.3595
21.3°G_{in}	7.5554	8.4344	8.8892	8.2553	7.8662	7.4590	6.9688	6.9483	7.1674	8.0098	7.9677	7.5070	92.9885
γ_{inc}	8.1074	8.9262	9.0926	8.9021	8.8634	8.2496	7.6629	7.7098	7.9029	8.3364	8.3279	7.9948	100.007
27° G_{inc}	7.6305	8.4063	8.9023	8.4907	8.1967	7.8731	7.0261	6.9975	7.2301	8.0763	7.8997	7.6817	94.349
γ_{inc}	8.1493	8.9212	9.1268	8.9764	8.8974	8.2855	7.7019	7.7447	7.9385	8.3742	8.3675	8.0364	100.52
α	0.352	0.323	0.374	0.41	0.43	0.487	0.565	0.486	0.541	0.41	0.323	0.322	

Table 7. Average values of global solar irradiance on an annual fixed orientation plane in El Jadida

Average values of global solar illuminance (Kwh/m^2) on a fixed annual orientation surface													
Inc (°)	Jan	Feb	Mar	Apl	May	Jun	July	Aug	Sept	Oct	Nov	Dec	G_{SA}
10° G_{inc}	4.1793	4.5952	5.8864	7.8852	8.8879	8.9599	8.3493	8.1723	7.5288	5.1220	3.9948	3.4528	77.5208
γ_{inc}	4.1821	4.9577	6.6872	8.8475	9.3306	9.5854	8.9366	8.4667	7.6375	5.5370	4.2497	3.6015	82.019
22° G_{inc}	4.6803	4.9748	5.9819	7.9870	8.9981	9.2534	8.8999	8.2806	7.7757	5.4459	4.2625	3.8318	80.3720
γ_{inc}	4.3711	5.3963	6.8544	8.9201	9.5079	9.9053	9.2670	8.7822	8.3403	5.7823	4.9385	3.7937	85.8591
27.4°G_{inc}	4.8562	4.9998	6.0362	8.1612	9.2234	9.8299	9.2006	8.7905	7.8645	5.8861	4.4360	3.9626	83.247
γ_{inc}	4.4401	5.4484	6.8998	8.9229	9.7638	10.021	9.3856	8.8956	8.7136	5.9704	4.9993	3.9989	87.4594
40° G_{inc}	5.1355	5.0032	6.0134	7.9997	9.5396	10.210	9.6658	8.2550	8.0116	6.0163	4.8475	4.0558	84.7493
γ_{inc}	5.5583	5.5100	6.9307	8.8572	9.9925	10.786	9.9989	9.0839	9.1014	6.7169	5.0805	4.2343	91.8592
45 G_{inc}	5.1933	4.9951	5.9465	7.8833	9.3269	10.102	9.3507	8.1275	8.0454	6.2782	4.9912	4.1989	84.4392
γ_{inc}	5.5875	5.5106	6.7136	8.8037	9.9066	10.562	9.8272	9.0026	9.1902	6.8802	5.1241	4.7454	91.8547
57° G_{inc}	5.2053	4.7958	5.6556	7.5578	8.8839	9.8888	9.1865	7.8887	8.0677	6.1436	4.7163	4.0318	82.0218
γ_{inc}	5.5799	5.4570	6.5055	8.6148	9.5278	10.299	9.6569	8.7799	9.1383	6.4740	5.0448	4.1283	89.241
α	0.448	0.470	0.448	0.323	0.340	0.323	0.360	0.374	0.330	0.448	0.360	0.560	

The maximum values of the histograms of the Figures 9 and 10, which correspond respectively to the optimal orientations of 27° and 40°, made it possible to draw the curves of the average monthly illuminations of Figures 11 and 12 for Garoua and El Jadida, respectively. The statistical indicators calculated for the curves in Figures 11 and 12 give us, respectively, (R= 0.9997, R^2= 0.9978, and MAPE= 4.1470) for the Garoua region, and (R= 0.9994, R^2= 0.9959, and MAPE= 7.7742) for the region of El Jadida. This again shows that the results obtained with the two methods are well correlated. Overall, we can say that flexibility and efficiency are the main great advantages of the two new formulas that we propose in (20) and (44), compared to the old equations (1), (2), and (34). Because, the latter, require many more climatic, geographical parameters or orographic data of the environment. Moreover, these old (1), (2), and (34) are very complex and require much more delicacy in their handling; hence the finding of an often high risk of error in the reconciliation to measured or expected values. In a precise way, we can say that the evaluation of the solar illumination on a plane and tilted surface based on the proposed formulas makes it possible to reduce the time of calculations, to minimize the errors of calculations, to increase the speed of convergence towards expected values, and the use of fewer measured parameters. Hence their advantages of being easily exploitable in several other fields of science.

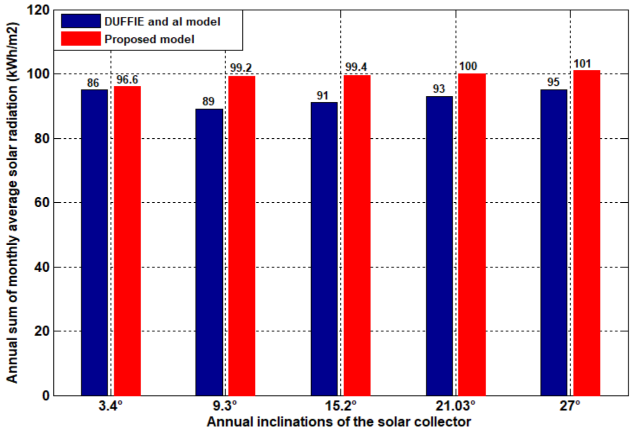

Figure 9. Histogram of annual sums of monthly average radiation according to the inclinations of the solar collector in Garoua

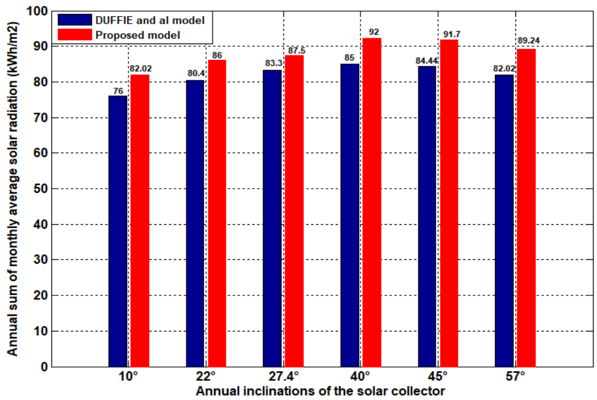

Figure 10. Histogram of annual sums of monthly average radiation according to the inclinations of the solar collector in El Jadida

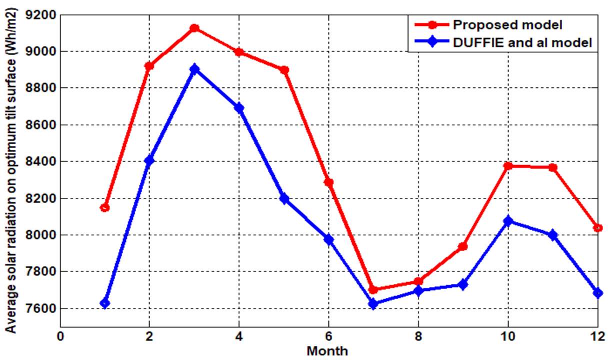

Figure 11. Average monthly solar irradiance according to the optimal inclination (27°) of the solar collector located in Garoua

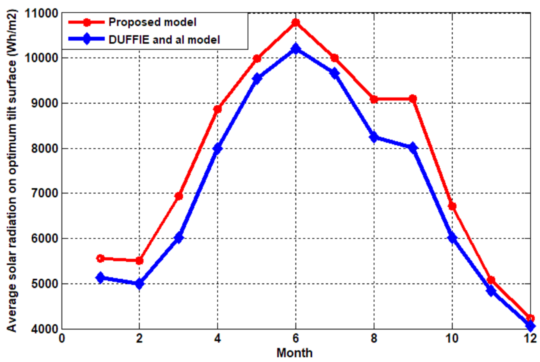

Figure 12. Average monthly solar irradiance according to the optimal inclination (40°) of the solar collector located in El Jadida

4. CONCLUSION

The very complex behavior of solar radiation leads us to improve the limits of previous models by proposing two new approaches to evaluating the solar deposit that are more efficient, flexible and better adapted to the climatic and orographic particularities of a place, guaranteeing better correlation between the estimated values and the measured values. The first new approach to evaluate or characterize the dispersion of global solar radiation in the sense of Joseph Fourrier's equation. This will result in a new formula for predicting solar radiation on a horizontal plane, and two characteristic formulas for global solar radiation in overcast and almost overcast skies. The second concerns a new approach for the evaluation of solar radiation captured by an inclined surface. This approach consists of associating solar radiation on a horizontal plane with the overall loss balance along the path of solar radiation. The validation of the proposed formulas was based on the analysis of statistical regression indicators using the calculated values and the data of effective sunshine and overall solar irradiation collected by measurements between the years 1991 and 1993 in the regions of Garoua in Cameroon and El Jadida in Morocco. It appears that, contrary to the results of classic models resulting from the Angstrom Prescott formula, the coefficients R= 0.9972, R^2= 0.9952, and MAPE= 0.061 for the Garoua data, and R= 0.8849, R^2= 0.9407, and MAPE= 0.05 for the El Jadida data, show that the results of the first proposed formula are well correlated with the measured values. Furthermore, we also note that using optimal tilt angles, the second proposed formula and the model of Duffie *et al.*, presented well correlated results, such as: R= 0.9997, R^2= 0.9978, and MAPE= 4.1470 for the Garoua data, and R= 0.9994, R^2= 0.9959, and MAPE= 7.7742 for the El Jadida data. Consequently, the formulas that we propose offer more advantages than other models, particularly in terms of the use of a reduced number of parameters, which makes it possible to reduce the complexity of implementation, reduce delays and risks of calculation errors. In addition to the better performance presented by the proposed models compared to the models cited in the literature, the proposed approach can be very beneficial and easily transposable to prospective research areas, as follows: i) the development of solar radiation prediction software associated with experimental emulators for the sizing of solar generators; ii) the study of the energy efficiency or energy balance of conventional or ecological buildings; iii) the study of the magnetic activity of the sun associated with the behavior of clouds which may be responsible for climate change; and iv) in agriculture with the calculation of ETP of plants.

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