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Three-phase power flow solution of active distribution network using trust-region method

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ABSTRACT

Distribution systems or networks are inherently unbalanced. As a result, single-phase power flow methods are generally no longer valid for such systems. Therefore, to obtain accurate results, unbalanced systems should be analyzed using three-phase power flow methods, which are far more complicated than the single-phase methods. Moreover, at present, the penetration of distributed generation (DG) in the distribution network has significantly increased. DG integration will increase the complication of the power flow analysis as it changes the network's basic configuration from passive to active system. This computational burden will significantly be higher if the power flow calculation has to be conducted several times (for example, in feeder reconfigurations or service restorations). This paper investigates the utilization of the trust-region method in obtaining the solution to the three-phase power flow problem of an active distribution network (i.e., distribution network embedded with DG). Trust-region computation algorithm is robust and powerful since the optimization technique is employed in finding new solutions in the iteration process. Results obtained from three representative unbalanced distribution networks (i.e., 10-node, 19-node, and 25-node networks) verify the validity of the proposed method. The effects of DG installation on distribution network steady-state performances are also investigated in the present paper.

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1. INTRODUCTION

The ability to provide valid information about system steady-state conditions at various loading and generation levels is necessary for the secure operation and proper planning of an electric distribution system. The information about system steady-state conditions is usually collected based on the results of power flow studies. The importance of this information can be explained as follows. For example, in the operational stage, if power flow results indicate that some electrical quantities violate their limits, then some corrective actions are needed to bring the quantities back to their allowable limits. This process needs a valid and robust power flow solution method for various system loadings, generations, and configurations.

Characteristics of an electric distribution network are usually different from those of an electric transmission network. These special characteristics of the distribution network make the conventional power flow solution techniques difficult or impossible to be applied [1], [2]. Moreover, the assumptions that the system is always balanced and can be analyzed using a single-phase model are also not applicable to a distribution network. The unbalance in a distribution network is usually of a higher level, and therefore cannot be neglected. Consequently, the single-phase model is no longer valid for the distribution system, and therefore, the system should be analyzed using a three-phase model. This will complicate the process of collecting information about system steady-state conditions, as three-phase power flow studies have to be used in the process [3]-[6].

At present, the penetration of distributed generation (DG) in the distribution network has significantly increased. In contrast to conventional power generation, which is usually large and connected to a transmission network, DG is a small-sized power generation and is generally connected to a distribution network [7]-[12]. Since DG is capable of supplying or injecting electrical power into the distribution network, it will change the network's basic configuration from a passive to an active system. This will also increase the computational burden in power flow studies. This computational burden will significantly increase if the power flow calculation has to be conducted several times (for example, in feeder reconfigurations or service restorations) [13]-[15].

Several methods for incorporating DG into power flow analysis have been proposed by some researchers [16]-[25]. In [16]-[20], node-based techniques, i.e., Newton-Raphson (NR) methods, have been used to find the solution to the power flow problem. The impact of the integration of renewable energy DG (i.e., wind turbine generator) in an electric network has been investigated in [16]. The power flow problem in [16] has been solved using the NR method. It has also been found in [16] that the integration of DG causes some voltage fluctuations, and the fluctuations can be minimized after the injection of reactive power. In [17], an approach named complex per-unit (CPU) normalization has been proposed. The proposed CPU normalization approach is intended to improve the performance of the NR-based method (i.e., fast decoupled power flow method) in solving the power flow problem of a distribution system. High penetration of DGs, more interconnection among distribution feeders, and distribution network high R/X ratios have also been considered in the method [17]. The development of an estimated solution to a set of nonlinear equations that represent electric power distribution networks has been discussed in [18]. The estimation in [18] has been made in terms of linear approximation of active and reactive power demands at PQ buses. The proposed model [18] can also be used as a tool for monitoring and controlling the design of the electric power distribution grid.

Sereeter *et al.* [19] explained that some new versions of the Newton power flow method for solving the three-phase power flow problem have been developed. The three-phase power flow problem is formulated in terms of power and current mismatch functions. To investigate the convergence property of the proposed method, different loading conditions and distribution network R/X ratios have been considered in [19]. The proposed method [19] has also been implemented and compared with the backward/forward sweep (BFS) algorithm for balanced and unbalanced electric power distribution systems. A power flow method special for reconfiguration analysis in a distribution system is proposed in [20]. The proposed method has been based on the extended fast decoupled method. A CPU normalization technique is also adopted in [20] to deal with the high R/X ratio networks. However, in many cases, the direct application of NR-based methods to distribution networks as proposed in [16]-[20] can cause convergence problems, and in turn, the solution is difficult or impossible to obtain. Although some techniques have been developed to improve the convergence of the methods, the NR-based methods still require a relatively long computation time [1], [2].

In [2], [21]-[25], branch-based techniques, i.e., BFS methods, have been employed to find the solution to the power flow problem of a distribution system. A power flow method that is derived based on the conventional BFS technique for an unbalanced distribution network is proposed in [2]. Unlike the conventional BFS, which employs two different steps (backward and forward sweeps) to calculate bus voltages, the method in [2] uses a single step to calculate the bus voltages. This single-step calculation is carried out through the use of the load-impedance matrix technique. In [21], an improved BFS power flow algorithm suitable for three-phase radial distribution systems is presented. In the method [21], Kirchhoff voltage and current laws are used in the backward sweep, and the linear proportional principle is employed in the forward sweep. The software package called distribution systems power flow analysis package (DSPFAP) for power flow analysis has been developed in [22]. Different BFS-based algorithms can be facilitated in the software package. Also, the effects of voltage-dependent load and DGs on power flow solutions can be investigated using the software package.

The result [23], a technique for handling PV nodes, i.e., nodes where DGs with constant voltage control are connected, is proposed. The technique [23] is based on loop analysis and incorporated in the FBS framework. It has been found in [23] that the convergence property of the proposed technique is satisfactory for a wide range of line R/X ratios and several connected PV nodes. The technique called quadratic-based BFS (QBBFS) method for distribution system power flow analysis is proposed in [24]. The method can accommodate multi-phase laterals, different load types, capacitors, distribution transformers, and DGs. Improvement to the conventional BFS power flow algorithm for unbalanced three-phase distribution networks has been proposed in [25]. In the algorithm [18], the network is represented as a tree-like structure, and therefore, the incidence matrix concept is not used in the algorithm. With this network tree-like structure representation, the use of redundant computations and the storing of unnecessary data can be avoided. However, BFS-based methods as proposed in [2], [21]-[25] require some difficult and special treatments in branch numbering and bus arrangement of the system. Moreover, the BFS-based methods are not designed to solve the power flow problem of meshed and/or looped distribution networks [1], [9], [17].

Recently, the trust-region method has been proposed as an alternative to the node-based techniques or the branch-based techniques for solving a set of nonlinear equations arising from the power flow problem formulation [1], [5], [9], [26], [27]. In [1], [5], the trust-region method has successfully been applied to obtain the power flow solution of a distribution network without DG. In [9], the trust-region method has also been applied in a balanced distribution network embedded with DG. This paper investigates the extension and application of the trust-region method to solve the power flow problem of an unbalanced distribution network embedded with DG (i.e., active distribution network). Results obtained from three representative distribution networks (i.e., 10-node, 19-node, and 25-node networks) are also presented in the present paper. The effects of DG installation on distribution network steady-state performances are also investigated and presented in the present paper.

The rest of the paper will be organized as follows. Section 2 discusses the formulation of the three-phase power flow problem of the distribution network. Section 3 addresses the formulation of the three-phase power flow problem of the distribution network embedded with DGs (active distribution networks). The unbalanced distribution power flow solution using the proposed trust-region method is explained in section 4. Case studies to evaluate the performance of the proposed trust-region method in solving the three-phase load flow problem of an active distribution network are given in section 5. Section 6 gives some conclusions of the present work.

2. THREE-PHASE POWER FLOW ANALYSIS OF DISTRIBUTION NETWORK

The relationship among the nodal quantities of a three-phase power system can be formulated. The formulation is expressed as follows [5], [28].

$$I^{abc} - Y^{abc}V^{abc} = 0 \tag{1}$$

In (1), I^{abc} and V^{abc} are vectors of nodal currents and voltages, respectively, and Y^{abc} is system admittance matrix. The vectors/matrix I^{abc}, Y^{abc}, and V^{abc} in (1) have the forms as follows:

$$I^{abc} = \begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ \vdots \\ I_n^{abc} \end{bmatrix}; V^{abc} = \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_n^{abc} \end{bmatrix}; Y^{abc} = \begin{bmatrix} Y_{11}^{abc} & Y_{12}^{abc} & \cdots & Y_{1n}^{abc} \\ Y_{21}^{abc} & Y_{22}^{abc} & \cdots & Y_{2n}^{abc} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}^{abc} & Y_{n2}^{abc} & \cdots & Y_{nn}^{abc} \end{bmatrix} \tag{2}$$

where:

$$I_i^{abc} = \begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}; V_i^{abc} = \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix}; Y_{ii}^{abc} = \begin{bmatrix} Y_{ii}^{aa} & Y_{ii}^{ab} & Y_{ii}^{ac} \\ Y_{ii}^{ba} & Y_{ii}^{bb} & Y_{ii}^{bc} \\ Y_{ii}^{ca} & Y_{ii}^{cb} & Y_{ii}^{cc} \end{bmatrix}; Y_{ij}^{abc} = \begin{bmatrix} Y_{ij}^{aa} & Y_{ij}^{ab} & Y_{ij}^{ac} \\ Y_{ij}^{ba} & Y_{ij}^{bb} & Y_{ij}^{bc} \\ Y_{ij}^{ca} & Y_{ij}^{cb} & Y_{ij}^{cc} \end{bmatrix} \tag{3}$$

Because of the characteristics of the electrical power system, the condition of each node is usually defined in terms of nodal power rather than nodal current. In terms of nodal power (and nodal voltage), the nodal current can be expressed as (4).

$$I^{abc} = \{[diag(V^{abc})]^{-1}(S_G^{abc} - S_L^{abc})\}^* \tag{4}$$

In (4), S_G^{abc} and S_L^{abc} are vectors of generation and load powers, respectively. The vectors S_G^{abc} and S_L^{abc} in (4) have the forms as follows:

$$S_G^{abc} = \begin{bmatrix} S_{G1}^{abc} \\ S_{G2}^{abc} \\ \vdots \\ S_{Gn}^{abc} \end{bmatrix}; S_L^{abc} = \begin{bmatrix} S_{L1}^{abc} \\ S_{L2}^{abc} \\ \vdots \\ S_{Ln}^{abc} \end{bmatrix} \tag{5}$$

where:

$$S_{Gi}^{abc} = \begin{bmatrix} S_{Gi}^a \\ S_{Gi}^b \\ S_{Gi}^c \end{bmatrix}; S_{Li}^{abc} = \begin{bmatrix} S_{Li}^a \\ S_{Li}^b \\ S_{Li}^c \end{bmatrix} \tag{6}$$

On using (4) in (1), the relationship among the nodal quantities of three-phase power system becomes (7).

$$\{[diag(V^{abc})]^{-1}(S_G^{abc} - S_L^{abc})\}^* - Y^{abc}V^{abc} = 0 \quad (7)$$

The set of nonlinear in (7) is the formulation of a three-phase power flow problem of an electrical power system in terms of nodal quantities. This set of nonlinear equations is then solved simultaneously to obtain the solution to the power flow problem. It is to be noted that a conventional distribution network usually has only one power source node or substation (SS) node, and the voltage at this node is normally specified. Because it has only one source node, power generations at the remaining nodes (load nodes) can be considered to be all zeros. Table 1 summarizes all of the known (specified) and unknown (to be calculated) quantities of the three-phase distribution system power flow formulation expressed in (7).

Table 1. Known and unknown quantities

Node	Known quantities	Unknown quantities
SS	$Y^{abc}; S_{Li}^{abc}; V_i^{abc} = \begin{bmatrix} 1\angle 0° \\ 1\angle -120° \\ 1\angle 120° \end{bmatrix}$	S_{Gi}^{abc}
Load	$Y^{abc}; S_{Li}^{abc}; S_{Gi}^{abc} = [0 \quad 0 \quad 0]^T$	V_i^{abc}

3. THREE-PHASE POWER FLOW ANALYSIS OF DISTRIBUTION NETWORK

It has been discussed earlier that penetration of DG in the distribution network will change the network's basic configuration from a passive to an active system. This change is expected since DG is capable of supplying or injecting electrical power into the distribution network. The first step in analyzing the power flow of the distribution network containing DG is to develop a proper model of the DG suitable for the power flow analysis.

In [9], [29]-[31], three models of DG for steady-state or power flow analysis have been proposed. The proposed DG models are as follows:
- Constant-PF model,
- Constant-Q model, and
- Constant-V model.

where PF, Q, and V stand for power factor, reactive-power, and voltage, respectively.

It has also been mentioned in [9], [29]-[31] that in power flow analysis, the nodes where DGs of Model 1 or 2 are connected can be considered as load nodes with negative power. Alternatively, they can be considered as generation nodes with constant active and reactive powers. On the other hand, the nodes where DGs of Model 3 are connected can be thought of as PV nodes, i.e., generation nodes with constant active powers and voltage magnitudes.

It is to be noted that the DG of Model 3 has the capability of regulating voltage magnitude and active power output. Since most of the DGs do not have these capabilities, only DGs of Model 1 or 2 are considered in the present work. However, if needed, DGs of Model 3 can also be included in the formulation in a straightforward manner.

From the above explanation, it is clear that formulation (7) is also applicable to a power system with DGs. However, the electrical quantities (known and unknown) in the formulation are slightly different. Table 2 lists all of the quantities in the power flow formulation of the distribution network embedded with DGs.

Table 2. Known and unknown quantities (active distribution network)

Node	Known quantities	Unknown quantities
SS	$Y^{abc}; S_{Li}^{abc}; V_i^{abc} = \begin{bmatrix} 1\angle 0° \\ 1\angle -120° \\ 1\angle 120° \end{bmatrix}$	S_{Gi}^{abc}
Load	$Y^{abc}; S_{Li}^{abc}; S_{Gi}^{abc} = [0 \quad 0 \quad 0]^T$	V_i^{abc}
DG	$Y^{abc}; S_{Li}^{abc}; S_{Gi}^{abc} = S_{DGi}^{abc}$	V_i^{abc}

4. POWER FLOW SOLUTION METHOD

As discussed in section 2, the formulation of the power flow problem (7) has the form of nonlinear equations. It is also acknowledged that the solution to this type of equation is usually obtained by using iterative techniques. NR method is probably one of the most popular iterative techniques for transmission network applications. Recently, another iterative approach, i.e., the trust-region method, has been proposed as an alternative to the NR method [1], [5], [9], [26], [27]. As also mentioned in [1], [5], [9], this optimization-based method is very suitable for distribution network applications.

This section briefly discusses the distribution power flow solution using the trust-region method. A more detailed explanation of the method can be found in [1], [5], [9]. Suppose that the general nonlinear equations to be solved are arranged in vector function form as (8).

$$F(x) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ f_2(x_1, x_2, \dots, x_n) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) \end{bmatrix} = 0 \tag{8}$$

Where x is the vector of unknown quantities to be calculated (or to be determined).

In the iterative approach, the vector x is calculated consecutively. The first step in the calculation process is to provide an initial estimation for x. This initial estimation is then iteratively corrected using (9).

$$x^{(k+1)} = x^{(k)} + d^{(k)} \tag{9}$$

Where k is the consecutive or iteration number and d is the vector of correction factors. The above calculation process is stopped when a solution with the desired accuracy has been obtained.

It is to be noted that in the NR method, the vector of correction factors d is determined directly by solving a set of linear equations. However, in the trust-region method, d is computed by using a constrained optimization technique as (10) [1], [5], [9], [26], [27].

$$\begin{aligned} &\min_{d^{(k)}} \left\| \frac{1}{2} h(x^{(k)}) + d^{(k)T} g(x^{(k)}) + \frac{1}{2} d^{(k)T} H(x^{(k)}) d^{(k)} \right\|^2 \\ &\text{subject to: } \left\| d^{(k)} \right\| \le \Delta^{(k)} \end{aligned} \tag{10}$$

Where $\Delta^{(k)} > 0$ is radius of the trust-region. It can be seen from (10) that a constraint is imposed on $d(k)$ to guarantee a valid solution to the vector $d(k)$. In other words, in the solution process, the vector $d(k)$ is confined within a certain region (trust-region). Also, in (10), h, g, and H are calculated using (11)-(13).

$$h(x^{(k)}) = F(x^{(k)})^T F(x^{(k)}) \tag{11}$$

$$g(x^{(k)}) = J(x^{(k)})^T F(x^{(k)}) \tag{12}$$

$$H(x^{(k)}) = J(x^{(k)})^T J(x^{(k)}) \tag{13}$$

Matrix $J(x)$ in (12) and (13) is widely known as the Jacobian matrix of $F(x)$. Elements of this matrix are determined using partial derivatives of the individual functions in $F(x)$. The formulation for the Jacobian matrix can be written as (14).

$$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{14}$$

It is to be noted that the NR method can sometimes fail to obtain a valid solution, especially when the Jacobian matrix is singular or ill-conditioned. The singular Jacobian matrix will complicate the computation of correction factors, and in many cases, it is impossible to be carried out. A different approach is employed to determine the correction factors in the trust-region method. It is determined using an optimization process and can handle cases where the Jacobian matrix is singular. In this way, the trust-region method can always provide a valid solution.

5. RESULTS AND DISCUSSION

5.1. Test systems

This section discusses case studies to evaluate the performance of the proposed trust-region method. In the study, the proposed method is used to solve the three-phase load flow problem of active distribution networks. The following three unbalanced distribution networks will be used to evaluate the performance of the proposed method.

i) 10-node distribution network [32], [33]

A single line diagram of this 8.66 kV system is given in Figure 1. Detail of line and load data can be found in [32], [33]. It can be seen from the load data that this 10-node network has a total load of 825 kW and 475 kVAR.

ii) 19-node distribution network [33]

A single line diagram of this 11 kV system is given in Figure 2. Detail of line and load data can be found in [33]. It can be seen from the load data that this 19-node network has a total load of 365.94 kW and 177.27 kVAR.

iii) 25-node distribution network [34]

A single line diagram of this 4.16 kV system is given in Figure 3. Detail of line and load data can be found in [34]. It can be seen from the load data that this 25-node network has a total load of 3,239.90 kW and 2,393.00 kVAR.

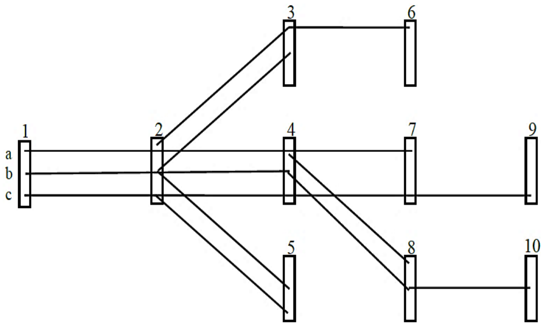

Figure 1. 10-node network

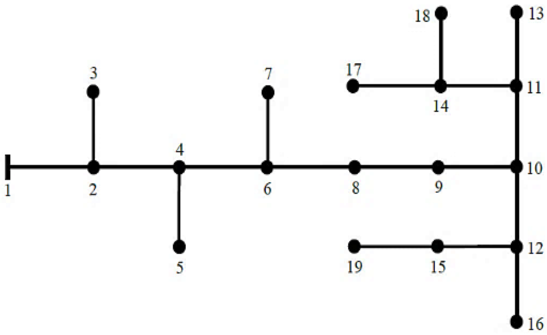

Figure 2. 19-node network

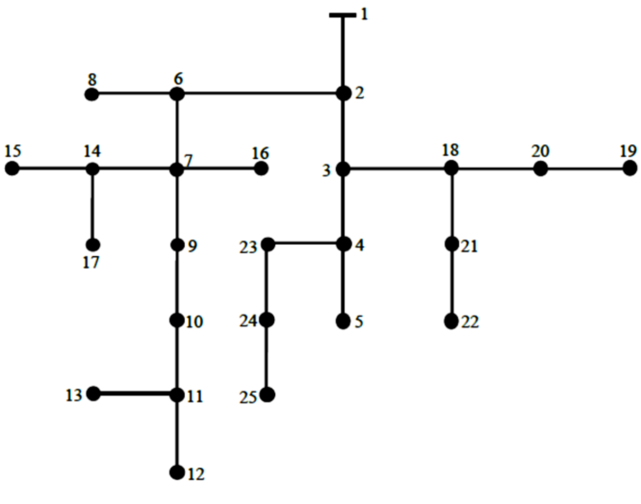

Figure 3. 25-node network

5.2. Power flow analysis results

In this section, the results of power flow studies for the three test systems described in subsection 5.1 are presented. Preliminary studies where DG is not included in the system are also carried out. These preliminary studies are intended to: i) validate the proposed trust-region method in solving the three-phase power flow problem of distribution networks; and ii) investigate the steady-state performance of unbalanced distribution networks without any DGs. The results of the studies, i.e., power flow calculations, in terms of voltage profiles, substation powers, and system losses for the test systems, are shown in Tables 3-6. By comparing the results in Tables 3-6 with those using the BFS method [32]-[34], the validity and accuracy of the proposed trust-region method are verified and confirmed. Tables 3-6 also show the results of power flow calculations for distribution networks embedded with DGs. Data on the DGs used in the calculations are presented in Table 7. It can be seen that the steady-state performances of the system with DGs are better than those of the system without DGs. A more detailed discussion of the comparison will be explained in subsection 5.3.

Table 3. Voltage magnitudes of 10-node network

Node	Without DG			With DG		
	Phase-a	Phase-b	Phase-c	Phase-a	Phase-b	Phase-c
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.95564	0.99303	0.98638	0.95970	0.99356	0.98770
3	0.94458	0.99295	0.98638	0.94869	0.99348	0.98770
4	0.93059	0.99165	0.97844	0.93854	0.99272	0.98106
5	0.95564	0.99063	0.98423	0.95970	0.99116	0.98555
6	0.91901	0.99295	0.98638	0.92323	0.99348	0.98770
7	0.92359	0.99165	0.97486	0.93160	0.99272	0.97749
8	0.92268	0.98995	0.97844	0.93070	0.99101	0.98106
9	0.92359	0.99165	0.96710	0.93160	0.99272	0.96976
10	0.92268	0.98154	0.97844	0.93070	0.98261	0.98106

Table 4. Voltage magnitudes of 19-node network

Node	Without DG			With DG		
	Phase-a	Phase-b	Phase-c	Phase-a	Phase-b	Phase-c
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.98746	0.98910	0.98798	0.98995	0.99126	0.99038
3	0.98542	0.98869	0.98633	0.98791	0.99085	0.98874
4	0.98235	0.98390	0.98301	0.98608	0.98714	0.98661
5	0.98201	0.98366	0.98283	0.98574	0.98690	0.98643
6	0.97928	0.98078	0.98005	0.98384	0.98474	0.98446
7	0.97861	0.98029	0.97956	0.98317	0.98425	0.98397
8	0.97281	0.97381	0.97347	0.97943	0.97957	0.97987
9	0.96592	0.96598	0.96574	0.97500	0.97388	0.97453
10	0.95625	0.95549	0.95500	0.96943	0.96696	0.96775
11	0.95499	0.95429	0.95330	0.96878	0.96629	0.96665
12	0.95478	0.95377	0.95358	0.96857	0.96578	0.96692
13	0.95440	0.95344	0.95210	0.96819	0.96545	0.96547
14	0.95449	0.95388	0.95282	0.96868	0.96623	0.96656
15	0.95274	0.95122	0.95126	0.96854	0.96498	0.96654
16	0.95339	0.95147	0.95217	0.96720	0.96351	0.96653
17	0.95365	0.95377	0.95232	0.96924	0.96692	0.96739
18	0.95380	0.95319	0.95209	0.96800	0.96554	0.96583
19	0.95159	0.94976	0.95047	0.96899	0.96492	0.96729

Table 5. Voltage magnitudes of 25-node network

Node	Without DG			With DG		
	Phase-a	Phase-b	Phase-c	Phase-a	Phase-b	Phase-c
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.97020	0.97110	0.97545	0.97563	0.97640	0.97976
3	0.96323	0.96444	0.96984	0.96870	0.96977	0.97418
4	0.95978	0.96219	0.96739	0.96527	0.96663	0.97173
5	0.95872	0.96025	0.96644	0.96422	0.96561	0.97079
6	0.95948	0.95587	0.96148	0.96579	0.96653	0.97060
7	0.94191	0.94283	0.94923	0.95808	0.95884	0.96314
8	0.95286	0.95378	0.95957	0.96369	0.96447	0.96871
9	0.93588	0.93668	0.94379	0.95473	0.95537	0.96010
10	0.93149	0.93186	0.93953	0.95299	0.95320	0.95821
11	0.92941	0.92963	0.93763	0.95248	0.95255	0.95772
12	0.92841	0.92839	0.93659	0.95342	0.95328	0.95863
13	0.92871	0.92872	0.93682	0.95180	0.95166	0.95693
14	0.93594	0.93699	0.94338	0.95477	0.95566	0.95968
15	0.93377	0.93487	0.94144	0.95417	0.95510	0.95914
16	0.94083	0.94177	0.94826	0.95702	0.95780	0.96219
17	0.93473	0.93595	0.94203	0.95359	0.95463	0.95835
18	0.95732	0.95864	0.96432	0.96282	0.96400	0.96867
19	0.95241	0.95443	0.95998	0.95794	0.95981	0.96435
20	0.95482	0.95634	0.96201	0.96034	0.96172	0.96638
21	0.95379	0.95487	0.96053	0.95932	0.96025	0.96491
22	0.95184	0.95246	0.95852	0.95738	0.95786	0.96290
23	0.95646	0.95838	0.96479	0.96198	0.96374	0.96914
24	0.95443	0.95651	0.96311	0.95996	0.96189	0.96747
25	0.95202	0.95469	0.96117	0.95756	0.96007	0.96554

Table 6. Substation power and network losses

System	Substation power (kW; kVAr)		Network losses (kW; kVAr)	
	Without DG	With DG	Without DG	With DG
10-Node	(853.96; 517.58)	(766.67; 476.38)	(28.96; 42.58)	(24.17; 34.63)
19-Node	(379.41; 183.07)	(300.09; 155.61)	(13.47; 5.80)	(7.33; 3.16)
25-Node	(3,390.02; 2,560.28)	(2,679.32; 2,162.10)	(150.12; 167.28)	(87.40; 104.12)

Table 7. DG data

System	DG capacity (kW; kVAR)	DG location
10-node	Phase a: 45.00; 21.00 Phase b: 20.00; 7.00 Phase c: 17.50; 5.25	Node 4
19-node	Phase a: 12.6330; 4.2861 Phase b: 11.6340; 3.9438 Phase c: 12.3270; 4.1790	Nodes 17 and 19
25-node	Phase a: 107.33; 55.44 Phase b: 108.33; 56.07 Phase c: 108.33; 56.00	Nodes 12 and 15

5.3. Discussion

The results obtained in subsection 5.2 are also presented in graphical forms (see Figures 4-8). These graphical forms are intended to better observe the effects of DG installation on the steady-state performances of the distribution systems. As clearly shown in Figures 4-7, the steady-state performances of the distribution systems are improved with the DGs installation. This improvement is demonstrated by: i) better voltage profiles (see Figures 4-6) and ii) lower network losses (see Figure 7). Figure 8 also shows another advantage of the DG installation. It can be seen that with DGs embedded in the system, the electrical power supplied by the distribution substation can be reduced. This result is expected since some loads are fed by the DGs.

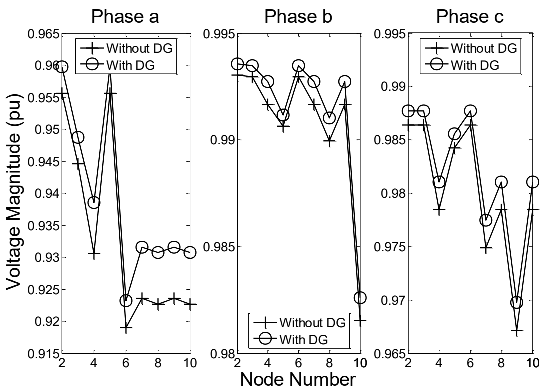

Figure 4. Comparison of voltage profiles of 10-node network

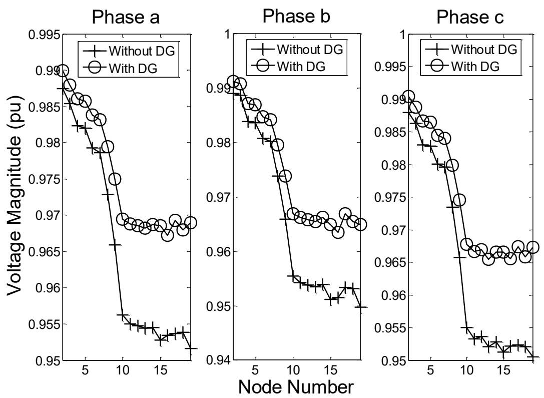

Figure 5. Comparison of voltage profiles of 19-node network

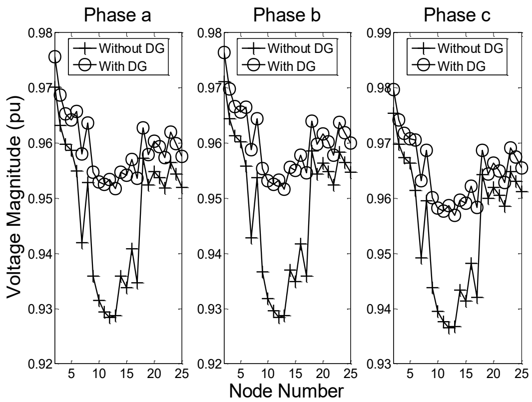

Figure 6. Comparison of voltage profiles of 25-node network

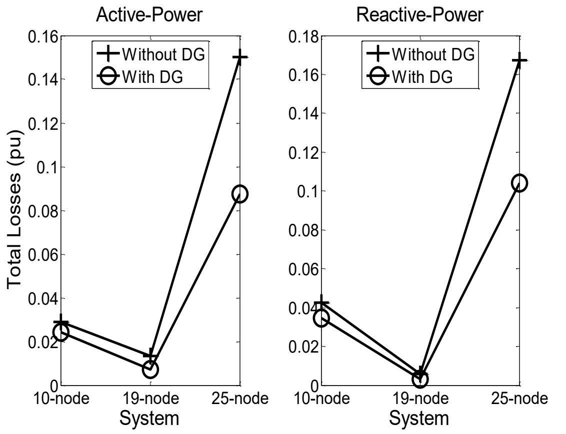

Figure 7. Comparison of network losses

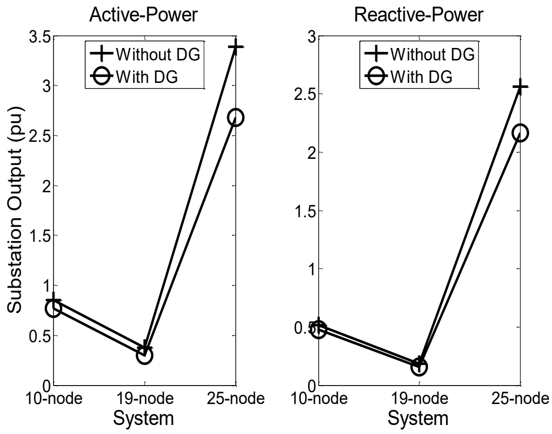

Figure 8. Comparison of substation powers

6. CONCLUSION

The application of the trust-region method in finding the solution to the three-phase power flow problem of the active distribution network (i.e., network embedded with DG) has been investigated in this paper. Trust-region computation algorithm is robust and powerful since the optimization technique is employed in finding new solutions in the iteration process. Another advantage of using the trust-region method is that some difficult and special treatments in branch numbering and bus arrangement are not required in the method.

Results obtained from three representative distribution networks (i.e., 10-node, 19-node, and 25-node networks) verify the validity of the proposed method. This conclusion is based on the excellent agreement between the results of the proposed method with those of the BFS method. The effects of DG installation on distribution network steady-state performances have also been investigated in this paper. It is confirmed that the system's steady-state performances improve with the installation of the DG.

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AUTHOR CONTRIBUTIONS STATEMENT

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Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
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M. Iqbal Arsyad		✓	✓			✓		✓	✓	✓	✓	✓		
Managam Rajagukguk		✓		✓		✓	✓			✓	✓		✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, [RG], upon reasonable request.

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