A-star algorithm based on admissible searching for strategically placing PMU considering redundancy and cost/benefit analysis

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ABSTRACT
This research examines an admissible search algorithm-dependent A-star strategy and takes into account redundancy and cost/benefit analysis under normal operating conditions. The goal is to allocate a phasor measurement unit (PMU) for maximal observability of the interconnected power network. To determine the fewest number of PMU required to make the connected power network totally observable using redundancy analysis, the A-star approach is utilized. The redundancy analysis of the power network is carried out in order to determine the appropriate PMU placement, which results in the acquisition of total power network observability and reliability. To put the suggested technique through its paces, it has been tested on IEEE-standard test systems such as IEEE-14 bus, IEEE-30 bus, New England-39 bus, IEEE-57 bus, and IEEE-118 bus. The results obtained using the suggested methodology are compared to those obtained through standard literature research. The experimental findings of the suggested method revealed the resilience and accuracy of the A-star algorithm as well as its effectiveness in achieving maximum observability of the connected power network.

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NOMENCLATURE
PMU : Phasor measurement unit
IEEE : Institute of Electrical and Electronics Engineers
SCADA : Supervisory control and data acquisitions
OPP : Optimal PMU placement
RCOs : Random component outages
MCDM : Multi-criteria decision-making
BPSO : Binary particle swarm optimization
ILP : Integer linear programming
ZIBs : Zero injection buses
RBs : Radial buses
RA : Redundancy analysis
CBA : Cost/benefit analysis
MR : Maximum redundancy

1. INTRODUCTION
Phasor measurement unit (PMU) is an online monitoring instrument in the wide-area measurement system, which is used for monitoring and supervisory purposes in leading and complex interconnected power networks. Till now, supervisory control and data acquisition (SCADA) is used for state estimation, but it provides asynchronized measurement dominant towards the unreliable analysis of power network conditions [1], [2]. In addition, the scan rate of data is 2-4 samples per cycle which makes the SCADA system
incapable of measuring dynamic/transient disturbances in interconnected power networks. PMU first came into existence in the mid-80s, which utilizes synchronized signals and provides synchronized measurements for actual time phasors of voltage and current. Synchronization is attained by the actual time sampling of voltage and current waveforms using timing signals from the global positioning system (GPS) clock [3], [4]. The observability of the interconnected power network is attained by the real-time data which is taken from PMU. PMU located at each bus in a connected power network efficiently observes all the states. But it is unwise and uneconomic to place the PMU at each bus in a power network as PMU and its communication services are expensive. Hence, an appropriate methodology is required for the optimal PMU placement problem (OPP) in the interconnected power network. The applicability of the present paper is restricted to the use of PMU for the complete observability of connected power networks. A connected power network is completely observable only when all of its conditions are evaluated separately, as given in [5], [6].

The appropriate site selection for placement of PMU in interconnected power networks has become a vigorous research issue [7], [8]. In recent years, various classical soft computing techniques and approaches have been published in the open literature for the optimal placement of PMU [9]. The authors propose a novel binary search algorithm-based methodology in [10] for the OPP problem for the complete observability of the interconnected power network states. Also, redundancy analysis is evaluated for the appropriate location of PMU. A novel three-step methodology for site selection of PMU using network connectivity data is proposed in [11]. The procedure considers PMU at each bus in a connected power network for observability of the states. Step 1 and Step 2 of the methodology decide: i) the removal of inappropriate bus locations of PMUs and ii) essential bus locations where PMUs are kept. Step 3 minimizes the number of PMUs using a pruning application. Abiri et al. [12], the ramification of the channel capacity of PMU on their site selection for total state estimation of the power network is suggested. Initially, the standard assessment of the grid network is formulated and then the revised methodology is indicated for the OPP problem. A genetic algorithm (GA) based approach is proposed in [13] for the OPP problem. It determines the minimum number and optimal locations of PMU and their geographical distribution for the full observability of the power network. The authors proposed an intelligent search-based approach for the OPP problem in [14]. Also, redundancy measurement is taken into consideration for minimizing the number of PMU. The authors proposed a novel methodology for the site selection of PMU suffering from random component outages (RCOs) in interconnected power networks [15]. Using the RCO model, the optimal PMU locations are preferred to minimize the state estimation error covariance.

Gómez and Ríos [16], the authors suggested two conditions for OPP issues. Firstly, an integer linear programming (ILP) based model for the OPP problem is presented. This methodology determines the minimum number and locations of PMU at each step while exaggerating the interconnected power network observability at each time. Secondly, a procedure is proposed to find the number of significant buses to be analyzed for dynamic/transient stability monitoring. An original method based on integer linear programming (ILP) and multi-criteria decision-making (MCDM) is provided in [17] to ensure the observability of power networks even during branch outages or PMU collapse. For the optimal location of PMU, ILP is used and then PMU location is prioritized by the proposed MCDM model. The authors propose a new investment decision model in [18] for the placement of PMU to ensure a complete estimation of the interconnected power network. Ramachandran and Bellarmine [19], a new meta-heuristic-based approach, namely the fruit fly optimization technique, is used to find the minimum number and location of PMU for the entire observability of the interconnected power network. A simple methodology for OPP problems in connected power networks using ILP is suggested [20]. A combination of graph theory and GA is used for the OPP problem to ensure state estimation of the power network [21]. Also, Theodorakatos [22], numerical observability formulation is used to determine the strategic locations of PMUs considering branch-and-bound and a binary-coded genetic algorithm. Further, a two-phase branch and bound are proposed in [23] to unravel the OPP problem. Later, a modified branch and bound are developed in [24] in order to solve the PMU placement problem.

Kavasseri and Srinivasan [25] deal with the joint OPP problem and power flow measurements to provide observability under faulted states in interconnected power networks. Initially, a non-linear integer programming technique is adopted to solve the problem, and then it is changed into an equivalent ILP problem through Boolean implications. Similar work is done in [26], in which both the stochastic and deterministic methods are used to find the fault location observability problem. Liao et al. [27], a hybrid two-phase approach is proposed for the OPP problem for a grid-connected power network. In the first phase, candidate locations of PMUs are found by using a graph theory approach. Then the local search heuristic approach is developed for finding the minimum number and optimal locations of PMU in the connected power network. A two-stage OPP method is proposed in [28]. In stage-1, the minimum number and locations of PMU to make grid network fully topologically observable and in stage-2, it is tested whether the obtained results of PMU locations lead to complete ranked measurement jacobian or not.
Xia et al. [29], an alternative approach for OPP with redundancy analysis for complete state estimation of the grid network conditions is proposed. A multi-objective biogeography-based optimization technique for OPP issues ensures that the interconnected power grid is fully observable [30]. In [31] and [32], the authors offered an ILP algorithm for an OPP problem to ensure complete observability of the connected power network states using tomlab optimization. Yazdel and Esmaili [33], a reliability-based approach is suggested for OPP problems and flow measurements. A GA-based method is used for the OPP issue in [34] and bus ranking formulation is developed for finding the minimal number of PMU necessary to make interconnected power networks entirely observable. A BPSO is recommended to solve the OPP problem [35], [36].

The purpose behind the whole research work is to propose a new method for solving the OPP problem. The researchers have adopted several techniques to solve the OPP problem, as stated in the literature survey. However, most of them are based on network topology and the evolutionary approach. The present paper has utilized the concept of admissible searching algorithm-based A-star approach for the OPP problem in order to determine the locations of PMUs, considering redundancy and optimally cost analysis. The main contributions of this present task are:

- To perceive the minimum number and optimal locations of PMUs to make the interconnected power network topologically observable.
- OPP methodology stated in this paper assures that the power system is entirely observable under normal operating states. In this work, the authors propose an A-star-based approach to find optimal locations of PMU for observability of power network states during normal operating conditions.
- Redundancy and cost analysis of PMU locations is attained by using the A-star technique in which the preferred final solution contains optimal locations of PMU for complete observability of the network states.
- The proposed approach is tested on IEEE 14-bus [37], IEEE 30-bus [37], New England 39-bus [38], IEEE 57-bus [37], IEEE 118-bus [37] test systems, and obtained results are compared with the standard referred journals published.

The structure of the paper is as follows: PMU placement rule is described in section 2. Section 3 explains the implementation of proposed approach to unravel OPP problem for the complete observability of the interconnected power network. In section 4, a discussion on case studies and results is performed. Section 5 concludes the paper.

2. PMU PLACEMENT RULES FOR INTERCONNECTED POWER NETWORK

The proposed OPP approach is applied to standard test systems so that PMUs can be placed at appropriate locations to estimate the connected power network states.

2.1. Methodology for optimal PMU placement (OPP) problem

The OPP issue can be prepared methodically as:

\[
\begin{align*}
\min & \sum_{k=1}^{N} c_k x_k \\
\text{s.t.} & \quad a(X) \geq b
\end{align*}
\]

Where, \( N \) is the number of buses in connected power network for placement of PMUs, \( c_k \) is the weight factor of estimation to the cost of placed PMU at \( k^{\text{th}} \) bus, and \( X \) is the binary decision variable vector having element \( x_k \) which decides attainability of PMU on \( k^{\text{th}} \) bus. Binary decision variable vector is explained by in (3) and \( a(X) \) is the observability constraint specific whose appearances are non-zero if the analogous different bus voltages are noticeable with respect to the given sets of measurements according to the above-mentioned rule or its appearances are zero otherwise.

\[
x_k = \begin{cases} 
1 & \text{if PMU is required at } k^{\text{th}} \text{ bus} \\
0 & \text{otherwise}
\end{cases}
\]

The entries in \( a \) are defined as:

\[
a_{k,i} = \begin{cases} 
1 & \text{if bus } k \text{ is connected to bus } i \\
1 & \text{if } k = i \\
0 & \text{otherwise}
\end{cases}
\]

And \( b \) is a unit vector, set as:

\[ a = (1, 1, \ldots, 1) \]
\[ b^T = [1 \ 1 \ 1 \ldots 1] \]  

(5)

### 2.2. Observability analysis based on PMU placement

When PMU is installed at a bus, it measures the voltage phasor at that bus and the current phasors of all the branches linked to that bus relying upon the number of channels. It is presumed in this work that PMU with an adequate number of channels is set up at a bus so that the current phasors of all the branches linked to that bus can be evaluated and voltage phasor of that bus too. The voltage phasors at the buses acquainting to the PMU setup bus can be resolved using the measured branch current phasors, bus voltage phasor and known line parameters [10]. Thus, with the optimal number of PMUs at a subset of power network buses, the complete estimation of the interconnected power network is attained. Figure 1 shows the observability of the connected power network with the help of PMU. As shown in Figure 1(a), if PMU is placed at B3, then it observes not only that bus but also all the acquainting buses, i.e., B1, B2 and B3 are connected to it. Assume if buses B2, B4 and including ZIB, B3, are observable by PMU, but bus B1 is unobservable, then B1 is observed by using (1) as shown in Figure 1(b). Suppose, if buses B1, B2 and B4 are observed by PMU but B3, a zero-injection bus is unobservable then B3 is measured using (1) as depicted in Figure 1(c). As shown in Figure 1(d), assume if B3 is a zero-injection bus, then PMU must be placed at ZIB to observe the B3 and its acquaintance buses B1, B2 and B4.

![Figure 1](image1.png)

Figure 1. Observability of interconnected power networks with the help of PMU: (a) PMU is placed at Bus 3, (b) ZIB observed by PMU, (c) ZIB is observed via KCL, and (d) PMU is placed at ZIB

In Figure 2(a), if PMU is placed at bus B2, then it observes B1, B2 (connected with PMU), B3 and B4 but B5, a radial bus (RB), is unobservable. Then, B5 is measured using (1). As depicted in Figure 2(b), if PMU is placed at a radial bus, then it observes only one bus B4 and itself. Therefore, the radial bus is not included in the placement of PMU.
3. **PMUs FOR COMPLETE OBSERVABILITY OF THE INTERCONNECTED POWER NETWORK**

PMU has now become the measurement technique of choice for connected power networks. The advent of PMU makes the power network fully observable during transient/dynamic states. The interconnected power network consists of a subset of buses i.e. substations, power generating units, or a huddle of loads and transmission lines connecting all the buses. PMU placed on power network buses gives the same-time evaluation of interconnected power network variables. This has been expedited with the help of GPS technology. A PMU established at a specific bus determines the voltage magnitude and phase angle of that bus and the current phasors of all branches connected to that bus rely upon the number of channels in an interconnected power network \[39\]. Accordingly, a power network is fully estimable only when all of its buses are observable over direct or indirect measurement. In the present work, the topological observability approach is used. Topological observability depends on graph theory methodology, where disoriented graphs illustrate the interconnected power network. An interconnected power network can be treated as topologically observable if not less than one mensuration tree is of full rank \[40\].

3.1. Modelling of zero injection buses

In an interconnected power network, some buses are neither connected with load nor with a generator, and such buses are termed as zero injection buses (ZIBs). ZIBs are modeled as in \[11\]. At ZIBs, a current is not injected into the connected power network. When ZIBs measurement is not taken into account for network observability, then pseudo-measurement is used for the connected power network observability. As depicted in Figure 3 buses are named as $B_1$, $B_2$, $B_3$, and $B_4$ where $B_3$ is represented as a zero-injection bus. By applying KCL at $B_3$, it gives the:

$$I_{13} + I_{23} + I_{43} = 0$$  \(6\)

3.2. Proposed approach

The minimum number and optimal locations of PMUs essential for full observability of the interconnected power network is found by using an admissible searching algorithm based on the A-star approach \[41\]. An A-star approach first came into reality by P. E. Hart, N. J. Nilsson, and B. Raphael of...

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Stanford Research Institute in 1968 [42]. It is a fortification of heuristic approaches like best-first search and the Dijkstra algorithm [43]. It is used in finding the shortest path. It is a probing-based approach like a greedy best-first search where a graph traversing algorithm observes the tiniest cost path from a stated initial node to the most promising node (out of one or more desirable goals) [43]. To solve the order in which the search visits nodes within the tree, it uses a distance-plus-cost node x (usually denoted by f(x)).

The distance-plus-cost probing is an addition of two functions:

\[ f(x) = g(x) + h(x) \]  

\[ g(x) = \text{The actual cost to a most promising node } x. \]

\[ h(x) = \text{Imprecise cost from node } x \text{ to goal node } x. \] It is a probing function. The testing function never considers overestimated costs, i.e., the actual cost of reaching a target node from node x should be \( > \) or = h (x). It is called the permissible probing approach. The absolute cost of each node is summed up as:

\[ f(x) = g(x) + h(x) \]  

A-star only bolsters the node if it appears most promising. It only targets to reach the goal node from the initial node and is incapable of getting each node. It is optimal if the probing function is permissible.

3.2.1. Implementation of the proposed approach

The key goal of the OPP issues is to render the device utterly measurable with a minimum number of PMUs. Thus, the node having higher connectivity of the branches should be favored for installing PMUs. This approach is used in the depth-first search (DFS) [44], but results give redundant PMUs also, which is unnecessary. The DFS is a graph theoretic procedure-based approach that searches each node in an interconnected power netw work leading to an inadmissible path to arrive at the goal node. The A-star approach overcomes this problem. To determine the fewest possible nodes for OPP, the A-star algorithm uses it to evaluate each node to decide which one should be augmented next [45]. The fewest possible nodes are recognized as those having minimum or maximum scores of an evaluation function. Although searching about the slightest optimal cost path, the A-star algorithm can alter its search path from the ongoing search path to the fewest possible way. This feature makes the A-star algorithm exceptional over other graph-theoretical procedures for OPP issues. Figure 4 shows the flowchart of the proposed A-star algorithm for the solution of the OPP problem.

![Flowchart of the proposed A-star algorithm for the solution of the OPP problem](image-url)
3.2.2. Step-by-step procedure

The implementation of the A-star approach to obtain the optimal location of PMU for a system consisting of \( N_b \) number of buses is given in the following steps:

- Read bus and line data of standard test system.
- With the help of given data, obtain binary connectivity matrix \( a( N_b \times N_b) \) as in (4). For example, in the IEEE 14-bus system, the connectivity matrix is obtained as:

\[
\begin{array}{cccccccccccccc}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]

- Create an initial binary connectivity matrix randomly and carry out the first set of estimations for \( g(x) \), \( h(x) \), \( cov \) and \( rocov \).
- Set ‘counter’ \( (c = c + 1) \). The counter is used to keep track of which row of each of the matrices we are on.
- Estimate the maximum ratios and save ratios values, \( r = \frac{h(cov) - h(0)}{g(x)} \) if \( r = 0 \), then delete the row so that it cannot be chosen again.
- Decrease the counter, ‘rocov_counter — 1’.
- Find the column index for each bus connection, ‘c_update’.
- Determine the number of columns of the \( cov \_find(y = size(cov \_find)) \), matrix for later use of doing a search through the \( cov \_find \) matrix.
- At each step of the ‘for loop’, take the column index and store 2 in the \( cov \_update \) matrix. Place 1 to represent that a PMU has been placed at \( k^{th} \) node. The 2’s are variables to show that the bus is covered.
- Check for less than optimal as:

\[
y^c = \begin{cases} 
2, & \text{if bus } i \text{ is fully observed by PMU,} \\
1, & \text{PMU has been placed at node } i \text{ where } y^c \text{ is the observability vector at } c^{th} \text{ iteration.} \\
0, & \text{otherwise} 
\end{cases}
\]

- Get the previous heuristic cost \( h(x) \) associated with placing the previous PMU ‘\( h(x) \_update, c \_update \)’.
- Compute the row and column index of each PMU placed in the last row of the \( cov \) matrix \( (\text{rowq}, \text{colw}) \).
- Obtain the number of PMU placed, ‘\( \text{cov} \_size \)’.
- Add a new row to \( g(x) \) whose value equals the number of PMUs placed times the number of buses in the system.
- Create a matrix containing the indices of all the connections to the nodes with PMU, ‘\( cov \_find \_additional \)’.
- Determine the number of buses on which PMUs are connected, \( N_{PMU} = size \) (as in step 14).
- \( N_{PMU} = (2, \text{delete row and column of the } \text{heu} \_\text{adj} \text{ that represents a covered node } 0, \text{otherwise where } N_{PMU} \text{ is the number of buses on which PMU are connected.} \)
- Sum up each column of \( \text{heu} \_\text{adj} \), so that it can be used to find the new row of \( h(x) \).

\[
h(x) > 0, \text{checks for nodes that still have the connection} \\
h(x) = h(x) \_update, \text{add the number of connection that node has to be the coverage value of the previous decision} \\
\]

- Make sure not to place same PMUs in a different order, such that \( h(x) \_update, c \_update = 0 \).
- For complete observability \( (\text{CO}_{PMU}) \) of the connected power network.

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$$\text{Display the PMU locations (} N_{PMU_L} \text{)}$$

3.3. Redundancy analysis

After determining the optimal locations of PMUs, the explanation for redundancy analysis (RA) can be presented as:

$$RA = \sum_{k=1}^{N_{PMU}} a N_{PMU_L}^T$$

Where $N_{PMU}$ is the total optimal number of PMUs, $a$ is the binary connectivity matrix obtained from (4), $N_{PMU_L}$ is the PMU locations at interconnected power network buses, which is obtained from step 19 of the proposed approach. In (8) allows the redundancy analysis for all the placement sets of optimal locations PMU.

3.4. Cost/benefit analysis (CBA)

In this section, the cost analysis associated with the optimal placement of PMU has been shown [46]. The cost analysis is formulated by (9).

$$CBA = \sum_{k=1}^{N_{PMU}} c_k N_{PMU_L}^T + FC$$

Where, $N_{PMU}$ and $N_{PMU_L}$ are explained in subsection 3.3. $c_k (=60,000)$ is described in subsection 3.1 and fixed charge (FC=$400,000) mainly refer to charge for hardware and software facilities.

4. RESULTS AND DISCUSSION

The optimization problem, described in the previous sections, is developed and executed in MATLAB software on an Intel Core i3 processor-based system with 2.2 GHz clock speed and supported by 3 GB of RAM. The proposed algorithm for the OPP problem is applied on standard IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, and IEEE 118-bus test systems. In the present work, $B_1, B_2, B_3, \ldots, B_n$ are defined as bus numbers where $n = 1, 2, 3, \ldots, N$.

4.1. Test system 1: IEEE 14-bus system

The IEEE 14-bus test system is taken into consideration as test system 1. The data of the system is shown in Table 1. The proposed algorithm is used to determine the OPP problem. Simulations are executed by ignoring and considering zero injection buses. Results are presented in Tables 2 and 3. The proposed method gives multiple solution sets for the OPP problem, as shown in Tables 2 and 3 ignoring and considering ZIBs. The IEEE 14-bus system has three optimal PMU placement sets with four optimal locations in each set ignoring ZIBs, i.e, \{ $B_2, B_7, B_{11}, B_{13}$ \}, \{ $B_2, B_6, B_8, B_9$ \} and \{ $B_2, B_6, B_7, B_9$ \}. For more than one solution, redundancy analysis is taken into consideration. The maximum value of redundancy analysis is taken as the best PMU placement set. Further, the installation cost is also taken into account for the placement of PMUs. As shown in Table 2, if PMU is placed at $B_2$, $B_6$, $B_7$, and $B_9$, then the maximum redundancy (MR) is 3.

Here $B_4$ is observed three times, $B_5$, $B_7$, and $B_9$ are observed two times and the rest of the buses observed once. In the IEEE 14-bus system, the maximum redundancy value for the first set is 16, for the second set is 17 and for the third set is 19 ignoring ZIBs. The set having maximum redundancy value is considered for PMU placement according to the proposed algorithm. So \{ $B_2, B_6, B_7, B_9$ \} is the best placement set for PMU ignoring ZIBs and \{ $B_2, B_6, B_9$ \} is the best location considering ZIBs. Bold letters give the most promising sites for the installation of PMUs.
the set with minimum redundancy value is considered for the best location for PMUs placement, ignoring ZIBs. From Table 7, the most promising locations to place PMUs optimally in the IEEE 30 bus system are shown in bold.

4.2. Test system 2: IEEE 30-bus system

IEEE 30-bus system is considered as test system 2. The information of the system is shown in Table 4. The proposed algorithm is used to determine the OPP problem in the IEEE 30-bus system. In test system 2, IEEE 30 bus system has three sets for PMUs placements. The first set is \{B_1, B_2, B_8, B_{10}, B_{12}, B_{18}, B_{23}, B_{27}\}, the second set is \{B_1, B_5, B_6, B_{12}, B_{17}, B_{19}, B_{22}, B_{23}, B_{27}, B_{28}\} and the third set is \{B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{24}, B_{25}, B_{27}, B_{28}\} ignoring ZIBs. The maximum redundancy value for the first, the second, and the third set is 34, 39 and 43 respectively as shown in Table 5. Hence, the third set is seemed to be for the placement of PMU. But, the set with a minimum number of PMU should be taken into consideration for the placement as the installation cost of PMUs is less. Further, the maximum redundancy is 3. Here one bus observed three times, three buses are observed three times and the remaining buses are observed once. In Table 6, eight locations in each set are found considering ZIBs. In this case, the set with maximum redundancy value is considered for placement of PMU as installation cost is uniform for each set. The most promising locations to place PMUs optimally in the IEEE 30-bus system are shown in bold.

4.3. Test system 3: New England 39-bus system

New England 39-bus system is taken as test system 3. The information on the test system is shown in Table 7. The proposed algorithm is used to determine the minimum number and optimal locations of PMUs for the new England 39-bus system. In Table 8, the set with maximum redundancy value 51 and minimum is 49 ignoring ZIBs. After considering the setup cost of placement of PMU, the set with minimum redundancy value is considered for the best location for PMUs placement, ignoring ZIBs. Again, in Table 9, the installation cost of PMUs is considered. Therefore, the set with minimum redundancy value is considered for the OPP considering ZIBs. Bold letters show the optimal locations for the New England 39-bus system.

| Table 2. Simulation results for the IEEE 14-bus system ignoring ZIBs |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| # of PMUs | Loc. of PMUs | # of times each bus observed | MR | RA | CBA($) | Comp. time in s. |
| 4 | B_2, B_5, B_{13}, B_{18} | 1 1 1 2 1 2 1 1 1 1 1 1 1 1 1 | 2 | 16 | 640000 | 0.0394 |
| 4 | B_2, B_6, B_8, B_{20} | 1 1 1 2 1 2 1 1 1 1 1 1 1 2 | 17 | 640000 | 0.0429 |
| 4 | B_2, B_6, B_8, B_{20} | 1 1 1 2 1 2 1 1 1 1 1 1 3 | 19 | 640000 | 0.0531 |

| Table 3. Simulation results for the IEEE 14-bus system considering ZIBs |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| # of PMUs | Loc. of PMUs | # of times each bus observed | MR | RA | CBA($) | Comp. time in s. |
| 3 | B_2, B_3, B_{12} | 1 1 1 1 1 1 1 1 1 1 1 1 1 | 2 | 12 | 580000 | 0.056 |
| 3 | B_2, B_3, B_{12} | 1 1 1 2 2 1 1 1 1 1 1 1 1 1 | 2 | 15 | 580000 | 0.091 |
| 3 | B_2, B_3, B_{12} | 1 1 1 2 2 1 1 1 1 1 1 1 1 1 | 2 | 15 | 580000 | 0.078 |

| Table 4. System data of IEEE 30-bus system |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| # of lines | # of ZIBs | # of RBs | Loc. of ZIBs | Loc. of RBs |
| 41 | 6 | 3 | B_6, B_8, B_{22}, B_{27}, B_{28} | B_{11}, B_{15}, B_{20} |

| Table 5. Simulation results for the IEEE 30-bus system ignoring ZIBs |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| # of PMUs | Loc. of PMUs | # of times each bus observed | MR | RA | CBA($) | Comp. time in s. |
| 8 | B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{23}, B_{27} | 1 2 1 1 1 2 1 1 1 1 1 1 3 1 1 1 1 | 3 | 34 | 880000 | 0.297 |
| 10 | B_1, B_5, B_6, B_{12}, B_{17}, B_{19}, B_{22}, B_{23}, B_{27}, B_{28} | 1 2 1 1 1 1 1 1 1 1 1 1 2 1 1 | 3 | 39 | 1000000 | 0.301 |
| 10 | B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{24}, B_{25}, B_{27}, B_{28} | 1 2 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 | 3 | 43 | 1000000 | 0.278 |

| Table 6. Simulation results for the IEEE 30-bus system considering ZIBs |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| # of PMUs | Loc. of PMUs | # of times each bus observed | MR | RA | CBA($) | Comp. time in s. |
| 8 | B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{23}, B_{27} | 1 2 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 | 3 | 32 | 880000 | 0.231 |
| 8 | B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{23}, B_{27} | 1 2 1 1 1 1 1 1 1 1 1 1 2 1 2 1 1 1 | 2 | 29 | 880000 | 0.242 |
| 8 | B_1, B_5, B_6, B_{10}, B_{12}, B_{18}, B_{23}, B_{27} | 1 2 1 1 1 1 1 1 1 1 1 1 2 1 2 1 1 1 | 2 | 29 | 880000 | 0.286 |

4.3. Test system 3: New England 39-bus system

New England 39-bus system is taken as test system 3. The information on the test system is shown in Table 7. The proposed algorithm is used to determine the minimum number and optimal locations of PMUs for the New England 39-bus system. In Table 8, the set with maximum redundancy value 51 and minimum is 49 ignoring ZIBs. After considering the setup cost of placement of PMU, the set with minimum redundancy value is considered for the best location for PMUs placement, ignoring ZIBs. Again, in Table 9, the installation cost of PMUs is considered. Therefore, the set with minimum redundancy value is considered for the OPP considering ZIBs. Bold letters show the optimal locations for the New England 39-bus system.
Table 7. System data of new England 39-bus system

<table>
<thead>
<tr>
<th># of lines</th>
<th># of ZIBs</th>
<th># of RBs</th>
<th>Loc. of ZIBs</th>
<th>Loc. of RBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>12</td>
<td>9</td>
<td>$B_1,B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$B_{20},B_{22},B_{23},B_{26}$</td>
</tr>
</tbody>
</table>

Table 8. Simulation results for the New England 39-bus system ignoring ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>3</td>
<td>49</td>
<td>11,20,000</td>
</tr>
<tr>
<td>13</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>3</td>
<td>51</td>
<td>11,80,000</td>
</tr>
<tr>
<td>15</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>3</td>
<td>51</td>
<td>13,00,000</td>
</tr>
</tbody>
</table>

Table 9. Simulation results for the New England 39-bus system considering ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>3</td>
<td>33</td>
<td>880000</td>
</tr>
<tr>
<td>8</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>2</td>
<td>33</td>
<td>880000</td>
</tr>
<tr>
<td>10</td>
<td>$B_2,B_4,B_6,B_8,B_{10},B_{12}$</td>
<td>$1231211111111111$</td>
<td>3</td>
<td>35</td>
<td>10,00,000</td>
</tr>
</tbody>
</table>

4.4. Test system 4: IEEE 57-bus system

IEEE 57-bus system is considered as test system 4. The data of the test system is given in Table 10. Using the proposed algorithm, the minimal number and optimal locations of PMUs for the IEEE 57-bus system are solved by ignoring and considering ZIBs. As shown in Table 11, the set with maximum and minimum redundancy values is 67 and 58. After considering the installation cost of PMUs, the set with minimum redundancy value is found suitable for the placement of PMUs ignoring ZIBs. The most promising locations for placement of PMU in IEEE 57-bus system ignoring ZIBs are shown in bold. In Table 12, the best PMU locations are $B_1,B_9,B_{20},B_{22},B_{23},B_{26},B_{28},B_{32},B_{35},B_{44},B_{50},B_{53},B_{56}$ after considering the installation cost of PMUs.

Table 10. System data of IEEE 57-bus test system

<table>
<thead>
<tr>
<th># of lines</th>
<th># of ZIBs</th>
<th># of RBs</th>
<th>Loc. of ZIBs</th>
<th>Loc. of RBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>15</td>
<td>--</td>
<td>$B_4,B_7,B_{11},B_{13},B_{14},B_{17},B_{20},B_{22},B_{26}$</td>
<td>$B_{28},B_{30}$, $B_{32},B_{35},B_{44},B_{50},B_{53},B_{56}$</td>
</tr>
</tbody>
</table>

*--* means not reported

Table 11. Simulation results for the IEEE 57-bus system ignoring ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>$B_2,B_4,B_6,B_{10},B_{12}$</td>
<td>$1111111111111111$</td>
<td>2</td>
<td>58</td>
<td>13,00,000</td>
<td>0.312</td>
</tr>
<tr>
<td>17</td>
<td>$B_2,B_4,B_6,B_{10},B_{12}$</td>
<td>$1222111112121111$</td>
<td>2</td>
<td>61</td>
<td>14,20,000</td>
<td>0.352</td>
</tr>
<tr>
<td>18</td>
<td>$B_2,B_4,B_6,B_{10},B_{12}$</td>
<td>$1111111111111112$</td>
<td>3</td>
<td>67</td>
<td>14,80,000</td>
<td>0.361</td>
</tr>
</tbody>
</table>

appropriate locations for those PMUs to be set up in order to provide complete observability of the number and optimal locations of PMUs.

The data for the IEEE 118-bus test system is given in Table 13. The promising locations for the placement of PMU in the IEEE 118-bus test system ignoring ZIBs are shown in bold letters in Table 14. In this test system, the maximum redundancy is 4. Here, one bus is observed four times, three buses are observed three times, sixteen buses are observed two times and the remaining buses are observed once. Table 15 is showing the optimal locations of PMUs considering ZIBs. In this case, only twenty-eight PMU is required in order to monitor the IEEE 118-bus test systems.

<table>
<thead>
<tr>
<th># of lines</th>
<th># of ZIBs</th>
<th># of RBs</th>
<th>Loc. of ZIBs</th>
<th>Loc. of RBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>10</td>
<td>7</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td></td>
</tr>
</tbody>
</table>

4.5. Test system 5: IEEE 118-bus system

The data for the IEEE 118-bus test system is given in Table 13. The promising locations for the placement of PMU in the IEEE 118-bus test system ignoring ZIBs are shown in bold letters in Table 14. In this test system, the maximum redundancy is 4. Here, one bus is observed four times, three buses are observed three times, sixteen buses are observed two times and the remaining buses are observed once. Table 15 is showing the optimal locations of PMUs considering ZIBs. In this case, only twenty-eight PMU is required in order to monitor the IEEE 118-bus test systems.

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>4</td>
<td>148</td>
<td>23,20,000</td>
<td>0.698</td>
</tr>
</tbody>
</table>

4.5. Test system 5: IEEE 118-bus system

The data for the IEEE 118-bus test system is given in Table 13. The promising locations for the placement of PMU in the IEEE 118-bus test system ignoring ZIBs are shown in bold letters in Table 14. In this test system, the maximum redundancy is 4. Here, one bus is observed four times, three buses are observed three times, sixteen buses are observed two times and the remaining buses are observed once. Table 15 is showing the optimal locations of PMUs considering ZIBs. In this case, only twenty-eight PMU is required in order to monitor the IEEE 118-bus test systems.

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>3</td>
<td>129</td>
<td>20,80,000</td>
<td>0.699</td>
</tr>
</tbody>
</table>

### Table 12. Simulation result for the IEEE 57-bus system considering ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>2</td>
<td>43</td>
<td>10,60,000</td>
<td>0.398</td>
</tr>
<tr>
<td>14</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>2</td>
<td>51</td>
<td>12,40,000</td>
<td>0.372</td>
</tr>
<tr>
<td>15</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 1 1 1 1 1 1 1</td>
<td>2</td>
<td>56</td>
<td>13,00,000</td>
<td>0.367</td>
</tr>
</tbody>
</table>

### Table 13. System data of IEEE 118-bus system

<table>
<thead>
<tr>
<th># of lines</th>
<th># of ZIBs</th>
<th># of RBs</th>
<th>Loc. of ZIBs</th>
<th>Loc. of RBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>10</td>
<td>7</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td></td>
</tr>
</tbody>
</table>

### Table 14. Simulation results for the IEEE 118-bus ignoring ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>4</td>
<td>148</td>
<td>23,20,000</td>
<td>0.698</td>
</tr>
</tbody>
</table>

### Table 15. Simulation results for the IEEE 118-bus system considering ZIBs

<table>
<thead>
<tr>
<th># of PMUs</th>
<th>Loc. of PMUs</th>
<th># of times each bus observed</th>
<th>MR</th>
<th>RA</th>
<th>CBA($)</th>
<th>Comp. time in s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>B₁₂, B₁₃p, B₁₃q, B₁₄p, B₁₄q, B₁₅p, B₁₅q, B₁₆p, B₁₆q, B₁₇p, B₁₇q, B₁₈p, B₁₈q, B₁₉p, B₁₉q, B₁₀p, B₁₀q, B₁₁p, B₁₁q, B₁₂p, B₁₂q</td>
<td>1 2 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>3</td>
<td>129</td>
<td>20,80,000</td>
<td>0.699</td>
</tr>
</tbody>
</table>

The proposed algorithm is carried out successfully on different standard test systems, and the obtained results are compared with the various existing methods proposed in referred journals in Table 16 (see Appendix). Based on the comparative analysis, it can be concluded that the proposed algorithm for OPP in the interconnected power network is more robust and efficient in terms of determining the minimum number and optimal locations of PMUs.

5. CONCLUSION

This paper presents a method for calculating the least number of PMUs to be installed and the appropriate locations for those PMUs to be set up in order to provide complete observability of the interconnected power network states under normal operating conditions. The A-star strategy, which is based on an admissible searching algorithm, has been developed for selecting the optimum site for the deployment of PMUs in order to provide comprehensive observability of the power network states. In the event that more

A-star algorithm based on admissible searching for strategically placing PMU ... (Rohit Babu)
than one solution is available, a redundancy analysis is proposed, and the installation cost is taken into account when determining the most promising locations for the allocation of PMUs. A successful application of the suggested approach has been demonstrated on conventional IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, and IEEE 118-bus test systems, and the results have been compared to those achieved using other previously published approaches. The obtained findings demonstrate the efficacy and robustness of the suggested OPP technique for interconnected power network observability.

**APPENDIX**

Table 16. Comparative analysis of obtained results with standard published literature

<table>
<thead>
<tr>
<th>Test System</th>
<th>14-bus</th>
<th>30-bus</th>
<th>39-bus</th>
<th>57-bus</th>
<th>118-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roy et al. [5]</td>
<td>4</td>
<td>10</td>
<td>13</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Marin et al. [6]</td>
<td>3</td>
<td>7</td>
<td>--</td>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>Venkatesh and Jain [8]</td>
<td>4</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Tai et al. [9]</td>
<td>4</td>
<td>--</td>
<td>13</td>
<td>--</td>
<td>32</td>
</tr>
<tr>
<td>Sodhi et al. [11]</td>
<td>4</td>
<td>--</td>
<td>15</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Mousavian and Feizollahi [12]</td>
<td>4</td>
<td>10</td>
<td>13</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Ramachandran and Bellarmine [13]</td>
<td>3</td>
<td>7</td>
<td>--</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>Gou [14]</td>
<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Zhao et al. [15]</td>
<td>--</td>
<td>7</td>
<td>--</td>
<td>11</td>
<td>29</td>
</tr>
<tr>
<td>Liao et al. [16]</td>
<td>3</td>
<td>--</td>
<td>8</td>
<td>11</td>
<td>28</td>
</tr>
<tr>
<td>Sodhi et al. [17]</td>
<td>4</td>
<td>--</td>
<td>15</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Xia et al. [18]</td>
<td>4</td>
<td>10</td>
<td>--</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Xu and Abur [22]</td>
<td>4</td>
<td>10</td>
<td>--</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Xu and Abur [23]</td>
<td>3</td>
<td>--</td>
<td>--</td>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>Ahamadi et al. [26]</td>
<td>3</td>
<td>7</td>
<td>--</td>
<td>13</td>
<td>29</td>
</tr>
</tbody>
</table>

Proposed method | 3 | 8 | 12 | 15 | 28 |

*"--" means not reported

**REFERENCES**


BIographies of Authors

**Rohit Babu** He was born in Mirzapur, India in 1986. He is currently working as an Associate Professor of Electrical and Electronics Engineering Department, Lendi Institute of Engineering and Technology, Vizianagaram (India). He had served the Department of Electrical and Electronics Engineering of Raffles University, Rajasthan (India), in 2013-2014, and Bharat Institute of Engineering and Technology, Hyderabad (India) for one year as an Assistant Professor in 2019-2020, respectively. He received his B.Tech. degree from United College of Engineering & Research, Greater Noida (India) in 2011 and M.Tech. Degree from Indian Institute of Technology (Banaras Hindu University), Varanasi (India) in 2013. He obtained his Ph.D. degree in Electrical Engineering from the Department of Electrical Engineering, Indian Institute of Technology (Indian School of Mines), Dhanbad, India in 2020. His research interests include power system observability, optimal PMU placement, state estimation, evolutionary approaches, power system optimization, reactive power planning, economic load dispatch and FACTS devices. He can be contacted at email: rohit.babu@lendi.org

**Biplab Bhattacharyya** He was born on 9th January 1970. He is currently working as an Associate Professor of Electrical Engineering at the Indian Institute of Technology (Indian School of Mines), Dhanbad, India. He has joined the electrical engineering department as assistant professor in the year 2007. Then he promoted to the post of associate professor in 2010. He had served the Department of Electrical Engineering of National Institute of Technology, Durgapur, India for six years as senior lecturer. He was in the position of lecturer in the Electrical Engineering Department of BITS, Pilani, Rajasthan, India for nearly one year. He worked as assistant engineer (electrical test), in the reputed cable industry for nearly three years. He obtained B.Sc (Hons) in physics from Calcutta University, India in 1990. He obtained his B-Tech and M-Tech degree in the field of electrical machines and power systems from Calcutta University in 1993 and 1995, respectively. He obtained his Ph.D. degree in engineering from the Department of Electrical Engineering, Jadavpur University, India in 2006. He has published several technical papers in international/national journals and conference proceedings. His research area mainly includes evolutionary approaches, power system optimization, planning, dispatch, FACTS devices. He can be contacted at email: biplabec@yahoo.in.