A 3D model of three phase shunt reactors by using a finite element technique with coupling to global quantities

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ABSTRACT

The finite element technique is used widely for researchers and manufacturers to design and simulate electrical systems in general and electrical machines such as shunt reactors (SRs) and transformers in particular. Many papers have recently applied several methods to analyze magnetic fields, copper losses and joule power losses in the shunt reactors (SRs). In this research, the finite element technique with coupling to global quantities is proposed to investigate the voltage and current distributions in the windings, and compute the distribution of magnetic field in the air gap and along the air core of the SR, as well as copper and core losses. The developed method is directly applied to the practical SR of 91 MVAr and a rated voltage of 500 kV. The finite element method (FEM)-simulated results are validated with experimental results to ensure accuracy and reliability. This facilitate designing the reactor.

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1. INTRODUCTION

Nowadays, the shunt reactor (SR) is widely applied to stabilize the electrical systems due to parasitic capacitance appearing in the transmission line. It is also applied to bound the short-circuit current and overvoltage in the electrical system. In addition, it can also be used to imbibe not only the reactive power created by a conductive capacitance in power transmission lines, but also adjust the reactive power in the electrical system. Recently, several studies have presented the many different method to investigate and analyse electromagnetic parameters of the SR [1]-[10]. In reference [1], the finite element method (FEM) was presented to investigate the flux fringing distribution around the air gap of the magnetic circuit of the SR. In reference [2], the theory of Maxwell stress tensor was proposed to evaluate the electromagnetic force on the core blocks causing the vibration and noise during the working of SRs. In reference [3], the paper investigated the influence of distance air-gaps on the iron-core of SRs by using the analytical method. In this work, the winding inductance was also computed to define the relationship of the distance between the air gaps. In references [4], [5], a mathematical model was studied to consider the nonlinear dynamic case. In references [6], [7], the relation between the SR and power system in various transient situations was investigated. In reference [8], the effects of core block numbers on the inductance was studied via the analytical technique. In reference [9], a FEM was presented to compute the magnetic flux density distribution and losses in the magnetic circuit of the SR. In reference [10], [11], the papers provided a useful tool for the
design and optimization of ferromagnetic-core inductors, which are commonly used in a variety of electronic devices. The analytical model presented can help engineers to reduce the time and cost involved in designing these devices while ensuring that the design meets the required specifications. In reference [12], this paper provides valuable insights into the use of air gaps in power inductor design. By understanding the influence of air-gap arrangements on inductor performance, designers can create more efficient and reliable inductors for a variety of applications.

It can be seen that the above studies have not investigated the voltage and current distributions on the windings, magnetic flux density on core blocks and in air gap between core blocks. In addition, the inductances of each winding phase has been not also considered in the previous studies. In this research, a FEM is developed to compute and design an oil-immersed SR of 91 MVAr, 50 Hz, 500 kV. The scenario of this paper is divided into two scenarios. In the first approach, an analytic model is developed to define the desired output parameters of the proposed SR. In the second approach, a FEM is developed with the obtained parameters in the first step to compute and analyse the voltage and current distribution, magnetic flux density distribution, Joule power loss density and winding inductances. The simulated results of the FEM are verified with the experimental results to validate on the proposed method.

2. ANALYTICAL APPROACH OF THE SR

One of the most important things in designing the SR is the factor of the air gap volume. This factor depends on parameters of the SR, i.e. reactive power, magnetic flux distribution in the magnetic circuit, winding inductances, power frequency, and energy stored in the air gaps and the windings. For the SR, the reactive power \( Q \) is determined as (1) [1], [10], [13].

\[
Q = U_{ef}I_{ef}
\]

Where \( U_{ef} \) and \( I_{ef} \) are respectively the effective voltage and current. It should be noted that the resistance of windings is small in comparison with inductances, therefore this value can be ignored. For that, the electromagnetic force (EMF) and effective current are determined as (2) and (3) [13]

\[
E_{ef} \approx U_{ef} = \left( \frac{2\pi}{\sqrt{2}} \right) \cdot f \cdot N \cdot B_m \cdot A_{gap}
\]

\[
I = \frac{N}{A_{gap}} \cdot \frac{B_m l_{gap}}{\mu_0 N}
\]

Where \( f \) is the frequency, \( N \) is the turn number, \( B_m \) is the maximum flux density, \( A_{gap} \) is the area of the air gap, \( l_{gap} \) is the length of the air gap, \( \mu_0 \) is the air permeability and \( R_{gap} \) is the air gap reluctance. Based on (2) and (3), the air gap volume \( V_{gap} \) can be presented via the air gap length \( l_{gap} \):

\[
V_{gap} = A_{gap} \cdot l_{gap} = \frac{Q}{\pi} \cdot \frac{\mu_0 B_m^2}{\mu_0}
\]

the air gap volume is considered as a constant if the quantity \( Q \), magnetic flux and frequency are the constant. Therefore, the dimension of core \( D_c \) is determined as (5) [14], [15].

\[
D_c = \sqrt{\frac{4A_g}{\pi}}
\]

The inductance of winding is calculated via the reactive power and voltage as (6).

\[
L = \frac{U_{ef}^2}{\omega Q}
\]

The size of the SR is influenced by the air gap length and area. Thus, the iron losses, copper losses and other factors depend on the different ratio value of the area and air gap length. Because this factor will decide to the cost and losses of the SR. The air gap permeance can be defined as (7) [16].

\[
P_{gap} = \left( \frac{A_{gap}}{l_{gap}} \right) \mu_0 = P_g
\]

The relation between the winding inductance and air gap permeance is: 

the winding inductance can also be computed via the expression as (9) [10], [13].

\[ L = \left( \frac{E_{\text{eff}}^2}{\omega f} \right) \]  \hspace{1cm} (9)

From (8) and (9), one gets (10).

\[ N = \frac{L}{\sqrt{\mu_0 \frac{\Delta_{\text{gap}}}{A_{\text{gap}}}}} = \frac{E_{\text{eff}}}{\sqrt{\omega f \mu_0 (\frac{\Delta_{\text{gap}}}{A_{\text{gap}}})}} \]  \hspace{1cm} (10)

3. **FINITE ELEMENT APPROACH**

3.1. **Canonical magnetodynamic model**

A model of a studied problem with a studied domain \( \Omega = \Omega_\Omega \cup \Omega_\Omega^c \) (where domain \( \Omega_\Omega \) is the conducting region and \( \Omega_\Omega^c \) the non-conducting region) is presented in Figure 1. The boundary of \( \Omega \) denotes \( \partial \Omega = \Gamma = \Gamma_h \cup \Gamma_e \). The set of Maxwell’s equations and behavior laws are written in the frequency domain as [10]:

\[ \text{rot } H = J_s \]  \hspace{1cm} (11)

\[ \text{rot } E = -j \omega B \]  \hspace{1cm} (12)

\[ \text{div } B = 0 \]  \hspace{1cm} (13)

\[ B = \mu H \]  \hspace{1cm} (14)

\[ J = \sigma E \]  \hspace{1cm} (15)

where \( H \): the magnetic field (A/m), \( B \): magnetic flux density, \( E \): electric field (V/m), \( J_s \): electric current density (A/m\(^2\)), \( J \): eddy current density (A/m\(^2\)), \( \mu \): relative permeability, and \( \sigma \): electric conductivity (S/m).

These fields defined in function spaces (\( H_h \) (rot; \( \Omega \)) and \( H_e \) (rot; \( \Omega \))) must satisfy Tonti’s diagram [10]. These function spaces contain the fields defined on \( \Gamma_h \) and \( \Gamma_e \) of the studied domain and BCs on \( \Gamma \) as (16) and (17) [10], [17]–[20].

\[ n \times H|_{\Gamma_h} = 0 \]  \hspace{1cm} (16)

\[ n \cdot B|_{\Gamma_e} = 0 \]  \hspace{1cm} (17)

Where \( n \) is the unit normal exterior to \( \Omega \). The set of (11), (12), and (13) is solved with the BCs in (14) and (15). In addition, global quantities with the current (\( I_i \)) and voltage (\( V_i \)) imposed the inductor are shown in Figure 2. The source of electromotive force (EMF) is defined through the surface \( I_{\Omega, i} \). i.e. (18) and (19) [8], [21].

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\[
\oint_{\Gamma_{1}} \mathbf{E} \cdot d\mathbf{l} = V_{i}
\]
(18)

\[
\oint_{\Gamma_{1}} n \cdot J \, ds = I_{i}
\]
(19)

Figure 2. Magnetodynamic model with global quantities (current and voltage) [10]

3.2. Magnetic vector potential weak formulations

The magnetic vector potential weak formulation \((B = \text{curl} \, A \text{ and } E = \sigma \partial_{t} A - \text{grad} \, \nu)\) is defined via the Ampere's law (10 A) in \(\Omega [10], [22]–[24].\)

\[
\left(\frac{1}{\mu} \text{curl} \, A, \text{curl} \, \nu'\right)_{\Omega} + (\sigma \partial_{t} A, \nu')_{\Omega} + (\text{grad} \, \nu, \nu)_{\Omega} \leq n \times H, \nu' >_{\Gamma_{h} - \Gamma_{t}}
+ \left< [n \times H]_{\Gamma_{t}}, \nu' \right>_{\Gamma_{t}} \leq \left< J, \nu' \right>_{\Omega} \forall \nu' \in H^{1}(\Omega)
\]
(20)

Notations of \((\., .)_{\Omega} \text{ and } <., . >_{\Gamma} \) are the volume and surface integrals, respectively, where the volume integral is defined in \(\Omega \) and the surface integral is defined on \(\Gamma. \) The function space \(H^{1}(\Omega)\) contains the test function \(w' \) and basis functions for \(A.\) At the discrete level, this function is determined by edge finite elements (FEs), which are a type of numerical discretization technique used to approximate the solution of partial differential equations. The boundary \(< n \times H, \nu' >_{\Gamma_{h} - \Gamma_{t}} \) in (20) represents the natural BCs of (15), typically zero. From (20), by taking \(w' = \text{grad} \, \nu' \) as a test function in the weak formulation (15), one gets (21).

\[
(\sigma \partial_{t} A, \text{grad} \, \nu')_{\Omega} + (\text{grad} \, \nu, \text{grad} \, \nu')_{\Omega} \leq [n \times H]_{\Gamma_{t}}, \nu' >_{\Gamma_{t}} \leq \left< J, \nu' \right>_{\Omega} \forall \nu' \in H^{1}(\Omega)
\]
(21)

Where \(\Gamma_{\bar{g}}\) is one of the surfaces \(\Gamma_{g,i}\), which is fixed by a current or voltage as shown in Figure 2.

3.3. Coupled to voltage quantity

The voltage applied \((V_{i})\) to the inductor can be expressed as a natural global constraint via an unit source electric scalar potential \(\nu_{s,i}\) associated with the applied voltage. The voltage applied to each passive inductor \((\Omega_{m,i})\) is expressed as a sum of the applied voltages between the electrodes \(\Gamma_{g}^{+}\) and \(\Gamma_{g}^{-}\). This results in an equation expressed as the circuit relation associated with the inductor \(\Omega_{m,i}\) via the voltage \(V_{i}\) [4], [25]

\[
\nu_{i} = \sum_{i} V_{i} \nu_{s,i}
\]
(22)

By substituting (17) into (16), one gets (23).

\[
\left(\frac{1}{\mu} \text{curl} \, A, \text{curl} \, \nu'\right)_{\Omega} + (\sigma \partial_{t} A, \nu')_{\Omega} + \sum_{i} V_{i} \left(\text{grad} \, \nu_{s,i}, \nu'\right)_{\Omega} \leq [n \times H, \nu' >_{\Gamma_{h} - \Gamma_{t}}
+ \left< [n \times H]_{\Gamma_{t}}, \nu' \right>_{\Gamma_{t}} \leq \left< J, \nu' \right>_{\Omega} \forall \nu' \in H^{1}(\Omega)
\]
(23)

3.4. Computation of Joule power losses via a post-processing

Joule losses are then computed with [21], [22]:

\[
P_{\text{loss}} = \frac{1}{2} \int_{\Omega} \frac{1}{\sigma} |J|^{2} \, d\Omega
\]
(24)

where \(J\) is the conjugate of \(I.\) By using the Poynting theorem associated to the surface integration of degrees of freedom of \(A\) and \(\nu\) located on the border, the volume integration can be defined as (25).
A 3D model of three phase shunt reactors by using a finite element technique ... (Hung Bui Duc)
Table 1. Parameters of the SR

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactive power</td>
<td>$Q$</td>
<td>91</td>
<td>Deep Yoke</td>
<td>$D_y$</td>
<td>776</td>
</tr>
<tr>
<td>Rated current</td>
<td>$I$</td>
<td>105</td>
<td>Total air gap length along the core</td>
<td>$l_g$</td>
<td>400</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>$U$</td>
<td>500</td>
<td>Number of turns</td>
<td>$N$</td>
<td>2244</td>
</tr>
<tr>
<td>Inductance</td>
<td>$X_L$</td>
<td>2712</td>
<td>Distance between the core and windings</td>
<td>$b_{cw}$</td>
<td>135</td>
</tr>
<tr>
<td>Core diameter</td>
<td>$D_c$</td>
<td>666</td>
<td>Yoke width</td>
<td>$W_w$</td>
<td>236</td>
</tr>
<tr>
<td>Core height</td>
<td>$H_c$</td>
<td>1978</td>
<td>Winding height</td>
<td>$H_w$</td>
<td>1559</td>
</tr>
</tbody>
</table>

The distribution of phase voltage in the windings is shown in Figure 5. It shows that the effective voltage applied to windings (phase A, phase B, and phase C) is the rated phase voltage with the value of 288,675 kV, which is close to the measured result of 288,7567 kV. This means that the significant error on the voltage simulation is less than 1%.

In the same way, the phase current distribution in the winding is presented in Figure 6. The comparison of current and voltage values between the simulated and measured results is given in Table 2, with errors being lower than 1% for both cases. The magnetic flux density distribution in the magnetic circuit is presented in Figure 7. It can be shown that the value is the biggest when the current of phase B is the maximum. The magnetic flux density distribution along the line of C1-C2 (core block) is presented in Figure 8. It shows that the magnetic flux mostly focuses on the surface of the core blocks, thus the magnetic flux density near edges of the core blocks is bigger than the one inside core blocks. In order to increase the leakage flux in the vicinity of the core blocks, the reluctance of magnetic circuit and number of air gap core blocks need to increase also.
Table 2. Comparison of simulated and measured voltages

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Rated values</th>
<th>Simulated results</th>
<th>Measured results</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current of phase A (A)</td>
<td>105</td>
<td>105.67</td>
<td>106.44</td>
<td>0.73</td>
</tr>
<tr>
<td>Current of phase B (A)</td>
<td>105</td>
<td>105.71</td>
<td>106.86</td>
<td>1.08</td>
</tr>
<tr>
<td>Current of phase C (A)</td>
<td>105</td>
<td>105.65</td>
<td>106.34</td>
<td>0.65</td>
</tr>
<tr>
<td>Voltage of phase A (V)</td>
<td>105</td>
<td>288.62</td>
<td>288.76</td>
<td>0.35</td>
</tr>
<tr>
<td>Voltage of phase B (V)</td>
<td>105</td>
<td>288.56</td>
<td>288.71</td>
<td>0.5</td>
</tr>
<tr>
<td>Voltage of phase C (V)</td>
<td>105</td>
<td>288.62</td>
<td>288.93</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 7. Distribution of magnetic flux density in the magnetic circuit

Figure 8. Distribution of magnetic flux density along the line of C1-C2 (core block)

The magnetic flux density distribution along the air gap (G1-G2) between core blocks is indicated in Figure 9. The results show that the magnetic flux density along the inner winding surface is very small and uniform due to the large number of air gaps. Its mean value is equal to 0.1797 T. The values of self and mutual inductances between phases (phase A, phase B, and phase C) are given in Table 3. The comparison of phase reactances between the rated and simulated results is given in Table 4. It can be seen that the error is lower than 1% for phase B and equal to 1% for phase A and phase C. In the same way, Table 5 shows the comparison of simulated and measured reactances for three phases. The errors on them are smaller than 2%. This is a good agreement with the developed method. In particular, it is satisfied the IEC 60076-6 of the SR. The copper and iron losses of the SR are pointed out in Figures 10 and 11, respectively. The results of iron and copper losses in the SR between the simulation and measurement are given in Table 6. The error is lower than 4% for the iron loss and 2% for the copper loss. These errors are completely acceptable and realiable.

Table 3. The values of self and mutual inductances (H)

<table>
<thead>
<tr>
<th>Inductance (H)</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase A</td>
<td>8.7257</td>
<td>-0.00565</td>
<td>-0.00129</td>
</tr>
<tr>
<td>Phase B</td>
<td>-0.00565</td>
<td>8.7199</td>
<td>-0.00566</td>
</tr>
<tr>
<td>Phase C</td>
<td>-0.00129</td>
<td>-0.00566</td>
<td>8.7268</td>
</tr>
</tbody>
</table>
### Table 4. Comparison of simulated and rated reactances

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Rated value (ohm)</th>
<th>Simulated result (ohm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactance (phase A)</td>
<td>2.712</td>
<td>2.739.08</td>
<td>1.00</td>
</tr>
<tr>
<td>Reactance (phase B)</td>
<td>2.712</td>
<td>2.735.88</td>
<td>0.88</td>
</tr>
<tr>
<td>Reactance (phase C)</td>
<td>2.712</td>
<td>2.739.42</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### Table 5. Comparison of simulated and measured reactances

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulated result (ohm)</th>
<th>Measured result (ohm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactance (phase A)</td>
<td>2.739.08</td>
<td>2.698.7</td>
<td>1.47</td>
</tr>
<tr>
<td>Reactance (phase B)</td>
<td>2.735.88</td>
<td>2.694.4</td>
<td>1.52</td>
</tr>
<tr>
<td>Reactance (phase C)</td>
<td>2.739.42</td>
<td>2.689.3</td>
<td>1.83</td>
</tr>
</tbody>
</table>

Figure 9. Magnetic flux density along the air gap (G1-G2) between core blocks

Figure 10. Copper loss in the winding of the SR

Figure 11. Iron loss in the magnetic circuit of the SR
5. CONCLUSION

In this research, the proposed SR was designed using an analytical approach to determine its main parameters. Afterward, a FEM simulation was conducted to simulate the prototype of the SR. The results of the SR obtained from the FEM were then compared to the that from the measured method to confirm their accuracy. The obtained results have also suggested that the analytical approach and FEM simulation are accurate methods for calculating various parameters of the SR. These parameters include voltage, current, inductance, reactance, copper loss, and iron loss. Additionally, these methods can also evaluate material costs, losses and other critical parameters. Moreover, the use of FEM simulation allows for a faster design and computing process and to optimize the prototypes needed to achieve the expected results. This approach can potentially save time and cost in the design and manufacturing of shunt reactors.

REFERENCES


Table 6. Comparison of simulated and measured copper and iron losses

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulated result (kW)</th>
<th>Measured result (kW)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron loss</td>
<td>68.519</td>
<td>70.817</td>
<td>3.24</td>
</tr>
<tr>
<td>Copper loss</td>
<td>152.377</td>
<td>155.321</td>
<td>1.90</td>
</tr>
</tbody>
</table>
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